

PHYSICO-CHEMICAL PROCESSES IN ICE AND FROZEN GROUND

**SIMULATING ICE ACCUMULATION AND DEFORMATIONS ON GROUND FREEZING:
A GENERALIZED THEORETICAL MODEL**

J.B. Gorelik

*Institute of Earth's Cryosphere, Siberian Branch of the Russian Academy of Sciences,
PO box 1230, Tyumen, 625000, Russia; gorelik@ikz.ru*

A single system of equations is suggested to simulate moisture transport responsible for segregated and injected ice accumulation and to calculate jointly the respective parameters and deformation in freezing ground. The system includes balance equations for heat and mass (water and soil skeleton) at the freezing front. Water fluxes for each mechanism are found with additional equations. The model allows predicting deformation patterns in surface foundations in the case of segregated ice accumulation and the percentage of mineral component in forming frozen structures for the case of injection. For the injection mechanism, there exists a critical water flux above which pure ice forms. The predicted parameters of injected ice accumulation agree with the field observations of the structure and composition of natural ice deposits, as well as with the results of specially designed laboratory experiments.

INTRODUCTION

Soil freezing is commonly attendant with redistribution of moisture below the freezing front, its accumulation in the frozen zone (in the form of various ice bodies which may reach quite big sizes), nonuniform deformation of freezing soil and loading on engineering structures with buried or surface foundations. Moisture flow in unfrozen soil which can cause stress and heaving may differ in origin, but those responsible for segregated and injected ice accumulation are considered to be most important [Kudryavtsev, 1978]. The segregation mechanism is explained in terms of surface and capillary forces near the boundary between the frozen and unfrozen portions of freezing soil [Beskow, 1947; Miller 1978; Grechishev et al., 1980; Gorelik and Kolunin, 2002]. These forces act like a pump which drives water upward to the freezing front. This mechanism (and segregated ice accumulation) most often works in very fine-grained soils, such as silt-size loamy sand and loam or clay, that freeze in open systems bordering open water. Ice accumulation by this mechanism produces stratified (or other) structures where pure ice lenses alternate with soil [Ershov, 1979]. Water fluxes, as well as the whole segregated ice accumulation process, depend largely on temperatures along the fringe of freezing soil, its mass transfer and thermophysical properties, and on external loading transferred to the base of the growing ice lens.

An exhaustive theoretical frost heave model was suggested by O'Neill and Miller [1985], and its simpler quasi-stationary analog was discussed in detail in

[Gorelik and Kolunin, 2002]. The model can account for the basic elements of the migration process (water fluxes, ice accumulation parameters, deformation, and depth-dependent changes in heaving stress) at the level of a laboratory sample. Yet, attempts of applying this knowledge to predict the effect on real engineering structures in freezing soil meet serious difficulties. For instance, the heaving stress on piles [Orlov, 1962] and surface foundations [Sazhin et al., 1984] increases with freezing depth according to field evidence but decreases in the case of laboratory samples [Gorelik and Kolunin, 2002]. This paradox is due to difference in ice lens loading: all load on a laboratory sample is transferred to the base of the growing ice lens while in real engineering structures it is redistributed following the deformation parameters of the freezing soil and the unfrozen ground beneath it.

The origin of injection flux is best understandable in the case of a closed volume freezing from above: a closed system without moisture input from outside. The freezing soil is fringed by impermeable walls and bottom (e.g., frozen soil or unfrozen dense clay) and consists of water-saturated coarse material (sand or debris). Rigid bonding between the freezing soil and the water-tight fringe reduces, possibly down to zero, vertical deformation along the walls. Because of the water-ice density difference, this produces high pressure (freezing pressure) in the water near the walls and a water flux from the freezing front inward the unfrozen soil. This flux (injection) is concentrated along the freezing front at the sites which are the

farthest from the confining walls and thus can heave under the freezing pressure. Zones of this heaving are at the same time the zones of excess ice accumulation if they are fed through the injection mechanism. The freezing soil adjacent to its rigid fringe can be called a zone of liquid moisture feeding (recharge) of the heaving zone.

Thus, appropriate simulation of the injection mechanism requires bearing in mind the deformability of freezing soil. Injected ice accumulation basically has no limitation from the soil grain size. According to data from large deposits of ground ice, most of them are of injection origin [Baulin et al., 1967]. Yet, until recently there were no theoretical models (nor reports of experimental studies) available in the literature for injected ice accumulation.

SYSTEM OF EQUATIONS IN THE MODEL AND ITS ANALYSIS

In [Gorelik, 2008, 2010] a single approach was suggested to solve the ice accumulation problem and to model the attendant mechanic effects for both mechanisms. The approach is based on balance equations at the freezing front, with an additional relationship between the velocities of the freezing soil boundaries:

$$j_h(H) = \kappa \rho_i n_f \dot{H}, \quad (1)$$

$$(n_f - n_u) \dot{z}_f = -(1 - n_f) \dot{z}_s, \quad (2)$$

$$\rho_w j_u(H) = \rho_i n_f \dot{z}_s + (\rho_w n_u - \rho_i n_f) \dot{z}_f, \quad (3)$$

$$\dot{z}_s - \dot{z}_f = \dot{H}. \quad (4)$$

Equations (1), (2), and (3) represent the heat balance and the mass balances of soil skeleton and water, respectively. The function $j_h(H)$ is the heat flux difference between the frozen and unfrozen portions of the ground, which depends on the frozen soil thickness H . When the temperature gradient $G(H)$ from the unfrozen zone is specified, the function $j_h(H)$ becomes

$$j_h(H) = \lambda_f \frac{t_f - t_s}{H} - \lambda_u G(H). \quad (5)$$

In (1)–(5) λ_f , λ_u , ρ_w , ρ_i , and κ are, respectively, the thermal conductivities of frozen and unfrozen soil, water and ice densities, and the heat liberated or consumed in the ice-water phase transition; t_s and t_f are the temperatures of freezing (on the ground surface) and phase transition (at the freezing front), respectively; n_u , n_f are the porosities of, respectively, unfrozen and frozen soil at the freezing front; z_s , z_f are the positions of the ground surface and the freezing front at a given time; $j_u(H)$ is the water flux from the unfrozen zone toward the freezing front (a function of H). The dotted variables (here and below) are the time derivatives. In the applied coordinates, the Oz axis is perpendicular to the freezing front and directed upward, and the unfrozen zone is fixed. The ve-

locities of the boundaries \dot{z}_s , \dot{z}_f are positive if they move upward and negative if they move downward.

Unlike the model by O'Neill and Miller [1985], the problem formulation in the suggested approach includes a single front: a freezing zone is absent and water accumulates at the freezing front. Furthermore, we use, for the first time, the mass balance equation for the soil skeleton (2) which allows describing ice accumulation in the freezing soil in terms of porosity (n_f). The latter is a continuous function of the freezing depth, unlike the discrete presentation of the structure parameters as in [O'Neill and Miller, 1985] which is quite cumbersome. The principal advantage of the new approach is the possibility, hard to effectuate otherwise, of formulating and solving problems for ice accumulation by both processes jointly with nonuniform deformation in freezing soil (with regard to external loading).

The main problem consists in specifying the water flux function $j_u(H)$ in (3) as a function of freezing depth with regard to the applied load, the freezing temperature, and the properties of specific soil. As the function has been specified, one arrives at a closed system of equations for four time-dependent unknowns H , z_f , z_s , n_f . The porosity of frozen soil n_f (depending parametrically on H) characterizes ice accumulation in the freezing soil and can be used to predict the deformation of the latter and other parameters of the process.

The function $j_u(H)$ can be either empirical or derived theoretically from water flux models. Before demonstrating the application of the suggested approach to specific problems, it is pertinent to dwell upon some general properties of system (1)–(4).

Successive removal of \dot{z}_s , \dot{z}_f , \dot{H} from these equations gives an equation for the frozen soil porosity $n_f(H)$ as related to heat and water fluxes ($j_h(H)$ and $j_u(H)$, respectively). The function $n_f(H)$ expressed via the dependence of the fluxes on the freezing depth is

$$n_f(H) = \frac{\rho_w n_u j_h(H)}{\rho_w n_u + \rho_i (1 - n_u) j_h(H) - \kappa \rho_w \rho_i (1 - n_u) j_u(H)}. \quad (6)$$

In the absence of water flux ($j_u(H) = 0$), the porosity n_f is depthward constant and coincides with that of soil frozen in a closed system [Gorelik, 2007]:

$$n_f(H) = \frac{\rho_w n_u}{\rho_w n_u + \rho_i (1 - n_u)} = \text{const.} \quad (7)$$

For the case of segregated ice, the model of O'Neill and Miller [1985] implies that the water flux $j_u(H)$ at any fixed H cannot exceed the maximum, unlike that in the stationary process, and satisfies the condition [Gorelik et al., 1998]

$$j_h(H) = \kappa \rho_w j_u(H). \quad (8)$$

Then, following (6) one obtains $n_f = 1$, i.e., pure ice forms, which is prerequisite to the ice lens growth in [O'Neill and Miller, 1985]. Furthermore, $\dot{z}_f = 0$ in this case, according to (2), i.e., the freezing front remains fixed relative to the underlying unfrozen soil while ice accumulation inside the freezing soil becomes fully transformed into upward motion (heaving) of the surface z_s . The value $n_f = 1$ being the limit porosity at the maximum water flux, the denominator in (6) is always positive and never zeroes at any intermediate flux value.

During injected ice accumulation, the deformation (and velocity) of the freezing soil surface is zero along its rigid fringe ($\dot{z}_s = 0$). The same happens in the ice segregation process at rather high stress on the freezing soil. In this case, (2) gives $n_f = n_u$, i.e., all excess water formed during segregation is forced away from the freezing front. The flow off the freezing front is found from (3) with regard to (4):

$$j_u = -n_u \dot{H} \Delta \rho_{wi} / \rho_w, \quad (9)$$

where $\Delta \rho_{wi} = \rho_w - \rho_i$. The flux is negative (inward the unfrozen soil) since $\dot{H} > 0$. Thus, the porosity of the forming frozen soil in the case of segregated ice accumulation is in the range $n_u \leq n_f \leq 1$, the lower and upper bounds corresponding, respectively, to the minimum (9) and maximum (8) fluxes.

In the case of injected ice accumulation, there is no upper constraint for water flux while the lower bound corresponds to the value defined by (9). The flux of (8) is critical for the injection mechanism. As the flux changes from minimum to critical, the porosity n_f changes monotonically from n_u to 1 (according to (6)), and the respective velocity of the freezing front \dot{z}_f remains negative but decreases monotonically in magnitude from the maximum (at $n_f = n_u$) to zero (at $n_f = 1$). Thereby ice soil forms with different relative percentages of ice and soil. When the flux exceeds its critical value, the velocity \dot{z}_f becomes positive, i.e., the freezing front moves off the fixed unfrozen zone, while the gap between this front and the unfrozen soil skeleton becomes filled with water; the porosities thereon become $n_f = n_u = 1$ and provide the second limit solution to (2). Physically it means that in the case of injected ice accumulation, at a supercritical water flux, the ice lens inside the freezing layer can grow in the presence of a continuously thickening water lens below it (until the water fluxes freeze up completely). Thus, the magnitude of water flux relative to its critical value should largely control the composition and structure of ice formed by the injection mechanism.

EXAMPLES OF MODEL APPLICATION

As it was shown in [Gorelik, 2008], the injected water flux during freezing of a closed soil volume de-

pends on the surface area ratio between the zones of recharge and heaving. Heaving in the freezing soil and the attendant pressure are estimated using an additional mechanic strain equation and the conservation law for a confined water mass. According to the calculated freezing front velocity \dot{z}_f and horizontal porosity (n_f) change, the predicted structure of injection-induced frost heaves corresponds to that observed in the field [Mackay, 1978]: an ice core encircled (in the map view) by ice-rich soil, with high ice contents decreasing away from the core; the ice core (nothing but the core) lies over a lens of clear water which gives an artesian flow when stripped (a subpingo water lens).

Laboratory physical modeling of injected ice accumulation [Gorelik, 2009] confirms the principal theoretical inferences: (1) ice soil with high soil percentages forms at below-critical injection water flux; (2) pure ice grows as water flux exceeds the critical magnitude. Furthermore, injected ice accumulation in the conditions of oscillating temperature patterns on the freezing soil surface produces rhythmically layered ice bodies or ice with sporadic lenses of coarse mineral material (Fig. 1).

The calculation procedure for the heaving and parameters of segregated ice accumulation was reported in [Gorelik, 2010]. For a surface foundation in the form of a flexible round disk with the radius R loaded uniformly with the stress σ , one estimates first deformation in the freezing soil and the response the latter receives from unfrozen foundation soil at any fixed H (Fig. 2). The freezing soil is treated as an elastic plate and the underlying unfrozen ground as an elastic halfspace. Note that modeling in [Gorelik, 2008] showed the principle possibility to include the rheology of frozen soil if the elastic solution is known. The deformation and the response are the functions of H , σ and the distance from the plate center r . Deformation in frozen soil occurs as settlement. The response is concentrated along the unfrozen-frozen soil boundary and, when taken with the opposite sign, defines the stress σ_R applied to growing ice lenses located in frozen soil in the immediate vicinity of the freezing front. It is noteworthy that for a laboratory sample, the stress σ_R is uniform and coincides with σ (the effect from the weight of its frozen portion is vanishing). In the model, however, this stress depends largely on the mechanic behavior of the frozen soil: if the latter is very stiff, σ_R will be independent of σ but depend uniquely on its gravitational pressure (which is often much less than σ). As a result, the predicted parameters of the laboratory sample and the natural structure behave in different ways.

After the stress σ_R has been estimated, one can find the water flux function $j_u(H)$ which likewise depends parametrically on σ and r . This function can be defined using a special software suggested in

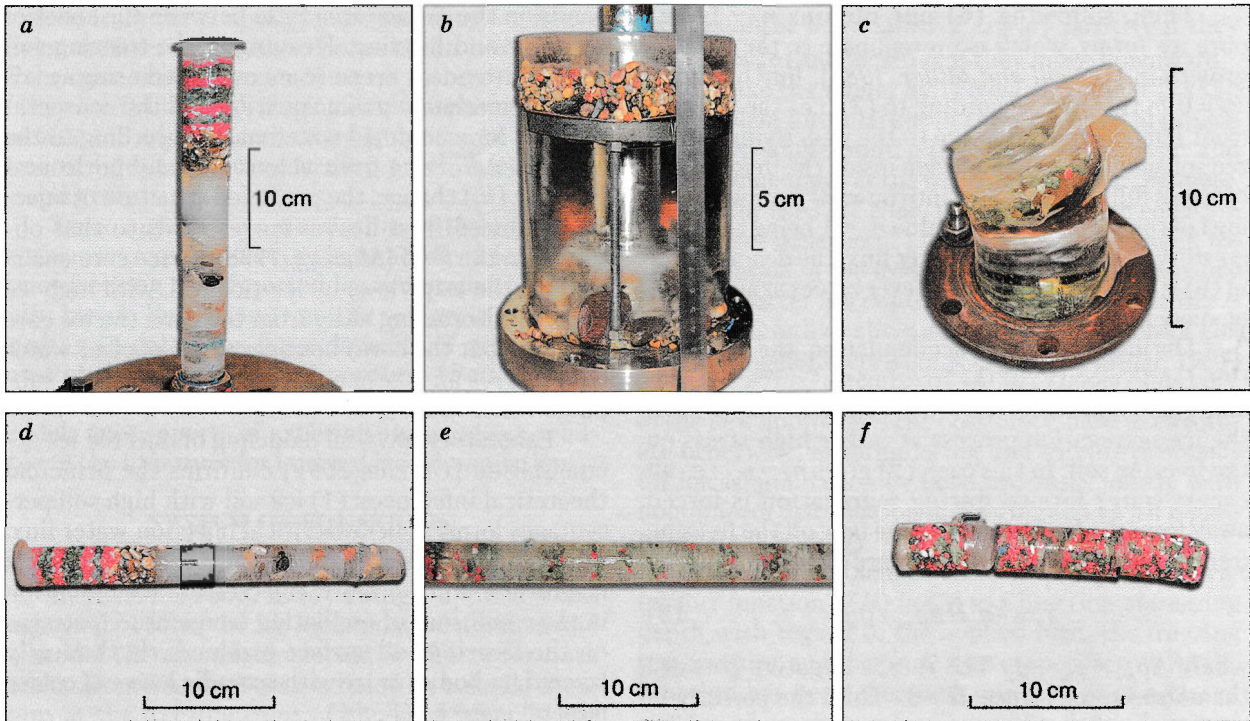


Fig. 1. Photograph of experiment results:

a – injection-fed frozen soil (deformation zone inside recharge zone); *b* – frozen soil in a feeding vessel; *c* – injection-fed frozen soil (deformation zone outside recharge zone); *d* – injection-fed frozen soil at above-critical water flux; *e* – rhythmically layered frozen soil at above-critical water flux; *f* – frozen soil (ice soil) at below-critical water flux.

[Gorelik, 2010], which is based on the scheme of O'Neill and Miller [1985]. The further procedure consists in obtaining the ice accumulation corresponding to the porosity n_f , heaving and total (heaving and settlement) deformation in a foundation using (1)–(4) as functions of the initial parameters. Thus, all model parameters depend on the freezing depth, the plate radius, the applied stress, and the soil properties. These dependences agree well, qualitatively and in the order of magnitude, with data of field experiments on deformation of surface foundations re-

ported in [Sazhin et al., 1984; Sazhin and Borshchev, 1987].

The model was applied to explain the behavior of pipelines in permafrost terrains. It was once suggested to reduce the thermal effect from gas pipelines on frozen soil and to increase the soil stability by means of gas cooling. This solution was partly put into practice but the result turned out to be ambiguous. There was reliable evidence of pipe segments emerging along the line even at pipe's negative buoyancy. This effect may be due to heaving stress from freezing soil on a buried structure. The main elements of the procedure of estimating deformation in freezing soil were applied to a buried pipeline with gas cooled to below 0 °C (Fig. 3). In the cases of Fig. 3, *a*, *b*, deformation is decaying and the position of the pipeline stabilizes inside the ground. However, the pipeline can be forced out to the ground surface after several seasonal freezing-thawing cycles when a frozen soil connection forms in cold seasons between the cold pipeline and the frozen soil (Fig. 3, *c*, *d*).

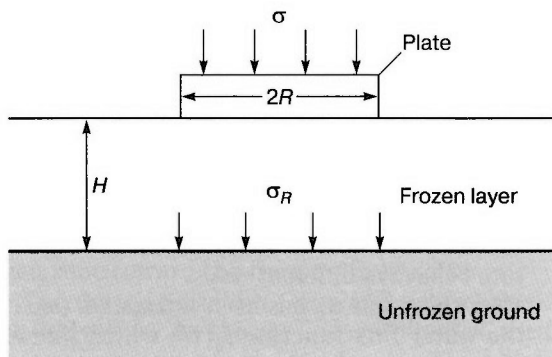


Fig. 2. Surface foundation.

CONCLUSIONS

The reported examples show that the suggested method for modeling ice accumulation jointly with related deformation in freezing soil allows explaining

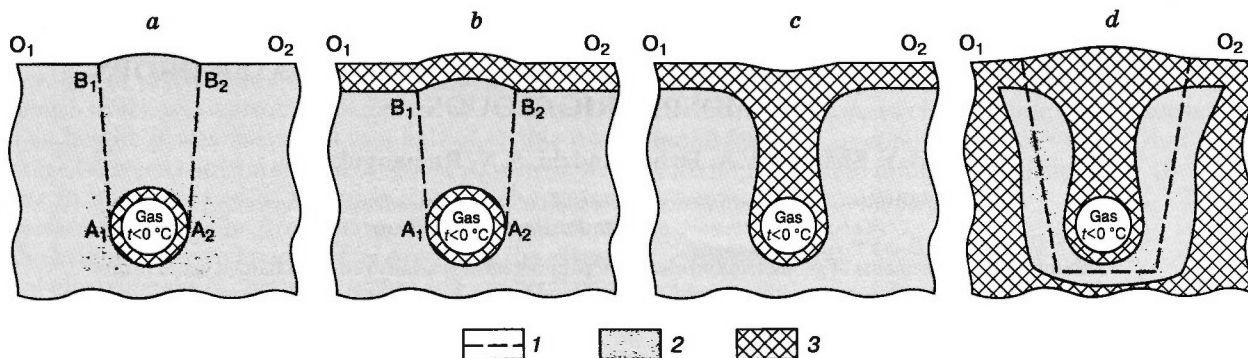


Fig. 3. Relative position of a cold pipeline and active layer (O_1O_2 is ground surface):

a – an unfrozen site in sporadic permafrost, warm season (A_1B_1, A_2B_2 – surfaces of unfrozen soil displacement); *b* – same in cold season, with disconnected frozen elements; *c* – same, with an ice-soil connection between pipe and frozen soil; *d* – a site of continuous permafrost, cold season, with an ice-soil connection: 1 – original trench profile; 2 – unfrozen soil; 3 – frozen soil.

important features of the processes at different mechanisms of moisture transport toward the freezing front. For the case of segregated ice accumulation, the problem to be formulated and solved is estimating heaving-induced deformation in elements of engineering structures. For the injection mechanism, the model can predict the amount of mineral component in ice-rich soil along the lateral extent of the freezing soil. In the latter case, there exists a critical water flux, with pure ice forming (in local zones of the freezing soil) above this critical value and ice-rich soil with high soil percentages forming below it.

This can provide explanation for the composition and structure of natural ice bodies and for specially designed laboratory experiments.

The new approach has a number of advantages over the existing methods based on discrete ice accumulation models. Further development of the method can be useful for geotechnical applications and analysis of geological processes.

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