MATHEMATICAL MODEL OF THE THERMOKARST LAKE PLAIN MORPHOSTRUCTURE UNDER CHANGING CLIMATIC CONDITIONS

A.S. Victorov, V.N. Kapralova, O.N. Trapeznikova

Sergeev Institute of Environmental Geoscience, RAS,
13, build. 2 Ulansky Lane, Moscow, 101000, Russia; dist@geoenv.ru

The aim of the paper is to provide a theoretical substantiation of and empirical support for a new model for morphological patterns developing in thermokarst lake plains in the context of climatic changes. The model is based on the approach of the mathematical morphology of landscape, using the random process theory. The research has resulted in the isometric lakes-based mathematical model for a morphological pattern of uniform thermokarst lake plains subject to climatic changes. It has been demonstrated analytically that the distribution of thermokarst lake areas should submit to the lognormal distribution, and their size distributions to Poisson distribution for different physiography and permafrost conditions in different regions, even under changing climatic conditions. These results agree with the empirical testing in key areas with different physiography, geology and permafrost conditions.

Morphological pattern, thermokarst lake plain, mathematical morphology of landscape, mathematical model

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INTRODUCTION

Thermokarst lake plains represent in a typical landform in many areas of Western Siberia, Northern Canada, and Alaska. The morphological structure of these landforms has been comprehensively investigated, with both quantitative morphometric aspects and thermokarst lake plains dynamics examined thoroughly. Some novel data and interesting results and findings have been obtained for lakes abundance and concentrations in these areas, and their size distribution [*Lastochkin, 1969; Burn and Smith, 1990; Fitzgerald and Riordan, 2003; Smith et al., 2005; Kirpotin et al., 2008; Dneprovskaya et al., 2009; Kravtsova and Bystrova, 2009*]. In this context, major research works focused on the identification of specific parameters measured in different areas, and their relations with geological, climatic and geocryological factors. Only few of them dealt with possible existence of some uniform patterns representative of a wide range of natural conditions, or aimed at developing appropriate models [*Victorov, 1995, 2006; Kapralova and Victorov, 2009; Polishchuk and Polishchuk, 2013; Kapralova, 2014*]. It should be noted that these models considered mostly the stable climatic conditions scenario. However, modeling the dynamics of such thermokarst centers complex is an essential element in the study of the entire picture of thermokarst processes. In practical terms, the model is solving the following tasks: 1) thermokarst processes – induced geological risk evaluation for linear structures; 2) predicting area dynamics affected by thermokarst lakes development; 3) preliminary quantitative estimates of geotechnical and geocryological conditions along the construction line.

LIMITATIONS AND ASSUMPTIONS OF MATHEMATICAL MODEL OF THERMOKARST LAKE PLAINS MORPHOLOGICAL STRUCTURE

The aim of this paper is to provide a theoretical substantiation of and empirical evidence for the new model of the developing morphological structure of thermokarst lake plains, which would take into account changes in climatic conditions.

The modeling was based on the mathematical landscape morphology approach [*Victorov, 1998, 2006; Kapralova, 2014*], which deals primarily with the study of a landform pattern produced by geographical landscapes on the Earth's surface. The random processes theory is widely applicable in the mathematical landscape morphology approach.

The model has been built for areas with homogeneous physiographic and geocryological conditions. By homogeneity we mean a combination of the following features: 1) the uniformity of surface topography, evidenced by their similarity in the satellite imagery; 2) constant material composition of the deposited surficial and underlying sediments; 3) surficial deposits thickness is persistent throughout the area; 4) the absence of buried ravines; 5) zero tectonic disturbances; 6) the entire area is confined to the same tectonic structure; 7) relatively homogeneous permafrost conditions. It should be emphasized that the model does not claim for overall uniformity, as it is focused on statistical homogeneity, allowing random variations.

The study area type is a slightly undulating subhorizontal plain dominated by various tundra vegetation (cotton grass tundra, sedge-cotton grass, etc.) and is interspersed with lakes with no developed ero-
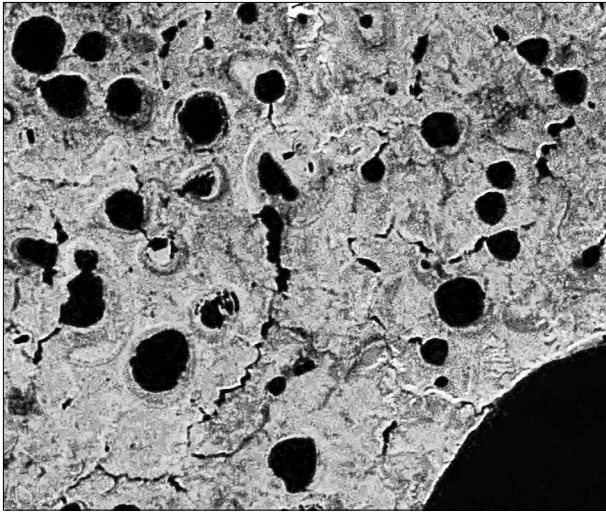
Fig. 1. A typical view of a thermokarst lake plain (the Yamal Peninsula) on satellite images (*Landsat* of 08.08.1999).

sional network. Lakes tend to have isometric, mostly rounded shape and are randomly scattered across the plain. A typical pattern of such plain is shown in Fig. 1. This landform type appears predominating in the areas of Western and Eastern Siberia, the Far East and the northern East European Plain (e.g., thermokarst lake plains account for at least 30 % within the permafrost zone of Western Siberia).

The model is thus not of generalized type and fails to encompass a significant number of incidences and aspects of thaw lakes development, for example, the distribution of lakes throughout the polygonal network created by ice wedges. In such cases, the morphological structure of an area is differentiated, e.g. lakes tend to be organized in geometric patterns along the network lines (orthogonal or any other), rather than distributed randomly.

When developing the model, we adhered to the mature ideas about thermokarst lake development utilized in a number of the permafrost studies (e.g., [*Perlstein et al., 2005*]). According to them, during its inception, the development of lake can be divided into two unequal stages. The first stage is associated with the origination of a topographic low, where a certain amount of water accumulates and the lake outlines are controlled by the preexisting relief, with the water column exceeding critical value. In this situation, thermokarst processes begin to develop within the topographic low (depression). This water body may be termed as thermokarst (thaw) lake only provisionally, given that according to the geomorphological principles, its denomination is derived from the morphology of the basin, hitherto unaffected by thermal abrasion.

In the second stage, lake's outline become circular or rounded, which implies that morphological features of a thaw lake are distinctly manifested in the examined homogeneous conditions [*Methodological Guidelines..., 1978*]. Finally, the stages of lakes coalescing may take place, succeeded by decaying thermoabrasive processes.

Given that the first stage is rather short-termed, the modeling is applicable to the second stage, which duration tends to be overriding, with key morphological features already distinctly marked. The model is based on the following assumptions:

– the process of thaw depressions development is probabilistic and it runs independently at non-intersecting sites, with probability of topographical lows inception within the sample area dictated by its extent alone (Δs). At smaller sites the probability for one topographic low (p_1) to form is much greater, rather than for multiple lows (p_k):

$$p_1 = \gamma \Delta s + o(\Delta s),$$

$$p_k = o(\Delta s), k = 2, 3, ...,$$

where γ is average number of lows per unit area;

– the growth of lakes sizes due to the thermoabrasive reworking occurs independently, which is directly proportional to the amount of heat stored in the lake and is inversely proportional to the lateral (sidewall) surface area of the lake basin.

Thus, a synchronous inception scenario will be considered, when primary lakes emerge over a relatively short time, as compared with the period of their development and, subsequently, the number of depressions remains constant. In this schematics, lake is assumed to be an object of cylindrical shape.

The first assumption drawn on the homogeneity of the studied area seem obvious, as it evidences the fact that there is only a finite quantity of thermokarst depressions (more specifically, their centers) in any limited area.

The second assumption also appears instructive, as it resolves itself to the expansion of a thermokarst foci (lake) size being proportional to the average heat transfer through the lateral surface of the lake basin.

Provided that such assumptions are simplified, they do not reflect the diversity of natural interrelations; however, the flexible nature of the assumptions derived from probabilistic approach provides a certain "safety factor".

Importantly, given that climatic characteristics are subject to change, the model does not provide for their constancy in time.

Analysis of mathematical modeling of thermokarst lake plains morphostructure

The model allows to analytically determine morphological patterns for thermokarst plains. As can be

shown [*Karlin, 1971*] by the assumptions underlying the model the number (k) of thaw depressions (centers) on a randomly selected area obeys the Poisson distribution [*Victorov, 1995, 2006*]:

$$P(k,s) = \frac{(\gamma s)^k}{k!} e^{-\gamma s},$$

where γ is average number of depressions per unit-area; s is sample area.

When considering resizing of thaw depressions, we divide the entire time period t into n intervals of length t_i, $i = 1, ..., n$, where environmental conditions can be assumed approximately constant.

Let's consider i-th interval $\left(\sum_{k=1}^{i-1} t_k, \sum_{k=1}^{i} t_k \right)$ and analyze the dynamics of one thaw depression within the i-th time interval.

The latter postulate underlying the model determines the proportionality of the so called random (stochastic) variation of the diameter of topographic lows over the j-th year of the considered time interval. Indeed, if V is the volume of water in the lake basin, which is a cumulative result of the amount of water contributed from the catchment area, evaporation, runoff and other processes of the water balance formation, then the lake bathymetry will be

$$h = \frac{V}{\pi u_j^2},$$

where u_j is the lake radius by the begin of the j-th year, whereas the area of inundated lateral surface is:

$$S = \frac{2V}{u_j}. \qquad (1)$$

Accordingly, the following expression is derived from the second assumption (with α as constant of proportionality)

$$\Delta u_j = \frac{\alpha c V}{S} \xi_j^0,$$

where Δu_j is size increment over the j-th year; c is specific heat; ξ_j^0 is random variable, reflecting the impact of incidental factors (precipitation during the year, precipitation mode, etc.), which after substituting the area size from the expression (1) and simplification takes the form of:

$$\Delta u_j = 2\alpha c u_j \xi_j^0.$$

Hence, after simplifying we obtain:

$$\Delta u_j = \xi_j u_j, \quad \sum_{k=1}^{i-1} t_k \le j \le \sum_{k=1}^{i} t_k, \quad \xi_j = 2\alpha c \xi_j^0. \qquad (2)$$

The proportionality coefficient ξ_j is a random variable allowing for the degradation progress within a specific time interval. This variable largely depends on the summer and winter temperatures, snow cover depth, the volume of storm runoff, soil temperature, precipitation type and amount, etc. The proportionality coefficients for different years are independent of one another, but equally distributed within the considered i-th time interval. We denote their mathematical expectation and variance as:

$$M\xi_j = a_i, \quad D\xi_j = \sigma_i^2.$$

When variables describing depressions sizes are shifted to the left hand side of the expression (2) and summed, we obtain:

$$\sum_{j=s}^{r} \frac{\Delta u_j}{u_j} = \sum_{j=s}^{r} \xi_j.$$

After substitution of the sum on the left with the integral we finally have:

$$\int_{x(s)}^{x(r)} \frac{du}{u} = \sum_{j=s}^{r} \xi_j, \quad \sum_{k=1}^{i-1} t_k \le s < r \le \sum_{k=1}^{i} t_k,$$

where $x(r)$, $x(s)$ is the extent of the spatial reduction at time points r and s, respectively, in the considered interval.

When the integration is performed, we obtain:

$$\ln x(r) - \ln x(s) = \sum_{i=s}^{r} \xi_i.$$

Given that the central limit theorem states that the sum of a larger number of independent variables (for example, [*Korolyuk et al., 1985*]) is normally distributed, this implies that at significant $(r - s)$ the thermokarst forms radii growth may be approximately regarded as the continuous time Markov chain with transition function

$$f(v,x,t) = \frac{1}{\sqrt{2\pi}\sigma_i x \sqrt{t}} \exp\left[-\frac{\left(\ln(x/v) - a_i t \right)^2}{2\sigma_i^2 t} \right], \quad (3)$$

where a_i, σ_i are distribution parameters in the considered time interval; v is initial size of thaw depression; x change in its extent over time t (within the i-th time interval).

Stating it otherwise, provided that thaw depression radius equals ζ_{i-1} by the begin of the analyzed time interval, then the following relation is true:

$$\zeta_i = \zeta_{i-1} \eta_i, \qquad (4)$$

where η_i is random variable equal to the ratio of the thermokarst form radius (ζ_i) at the end of the time interval to the radius (ζ_{i-1}) at the beginning of the considered (i-th) interval. This value, as follows from

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the relation (4), has a lognormal distribution with the following parameters

$$M[\ln\eta_i] = a_i t_i, \quad D[\ln\eta_i] = \sigma_i^2 t_i, \quad i = 1, ..., n-1. \quad (5)$$

Given that the initial size of thermokarst form provisionally taken as 1, the following relation can be readily obtained from (4)

$$\ln\zeta(t) = \ln\eta_1 + \ln\eta_2 + ... + \ln\eta_{n-1} + \ln\eta_n,$$

$$\sum_{i=1}^{n-1} t_i \le t \le \sum_{i=1}^{n} t_i,$$

where $\zeta(t)$ is thermokarst form size at the time t; η_i ($i = 1, ..., n-1$) are independent lognormal random variables with the parameters specified in (5); η_n is an independent lognormal random variable with the parameters

$$M[\ln\eta_n] = a_n\left(t - \sum_{i=1}^{n-1} t_i\right),$$

$$D[\ln\eta_n] = \sigma_n^2\left(t - \sum_{i=1}^{n-1} t_i\right).$$

Hence, taking into account the summation of independent normally distributed variables, it follows that the thermokarst form size logarithm at the end of the time interval $(0, t)$ is a normally distributed random variable with mathematical expectation

$$M[\ln\zeta(t)] = \sum_{k=1}^{n-1} a_k t_k + a_n\left(t - \sum_{k=1}^{n-1} t_k\right)$$

and variance

$$D[\ln\zeta(t)] = \sum_{k=1}^{n-1} \sigma_k^2 t_k + \sigma_n^2\left(t - \sum_{k=1}^{n-1} t_k\right).$$

Thus, even in case of climatic changes the distribution of radii, and therefore areas of thermokarst forms obey the lognormal law with linear but not piecewise-linear growth parameters.

This implies that for any time we get a lognormal distribution of the thermokarst forms diameter:

$$F_d(x,t) = \Phi\left(\frac{\ln x - at}{\sigma\sqrt{t}}\right),$$

or, for probability density function

$$f_d(x,t) = \frac{1}{\sqrt{2\pi}\sigma x\sqrt{t}}\exp\left[-\frac{(\ln x - at)^2}{2\sigma^2 t}\right],$$

where $F_d(x, t)$ is diameter distribution; $\Phi(t)$ is Laplace's function; a, σ are distribution parameters; t – time passed from the onset of the process.

It is easy to show that in case of smooth change of climatic parameters ($a(t)$, $\sigma(t)$) the result is identical, but the expressions for mathematical expectation and variance have been subject to a slight change:

$$M[\ln\zeta(t)] = \int_0^t a(u)du, \quad D[\ln\zeta(t)] = \int_0^t \sigma^2(u)du.$$

With regard to the considered simulations, it is not essential, whether the shape of lakes is circular in the plan, assuming that in the course of thaw lake expansion the geometric similarity of the outlines is preserved. In this case, the lake has a cylinder shape, however its guideway shape will be arbitrary, rather than circular.

The model is not to be envisaged as encompassing all possible scenarios. For example, in Yakutia, we have encountered a very different morphology represented not by isometric but festoon form of thaw lakes, which appears the case that requires separate examination and analysis.

It also appears essential, to what extent the proposed model is affected by the ongoing thermoabrasion processes. In other words, what happens if at some time point the processes activity has dropped sharply, with the growth of lakes outlines in the plan considered temporarily ceased? The situation described above is similar to the one, when due to the influences of climate change in the area the growth of lakes is almost blocked out, and then as the conditions become again favorable for thermal abrasion, the processes revive.

These questions can be answered using the model described. Let's consider the situation with the resumed growth processes breaks. We denote the time stop interval as t_i. Due to the breaks the following relations are true for this interval:

$$a_i = 0, \quad \sigma_i^2 = 0,$$

and, consequently, we deal with individual case modelling, namely for the given i

$$\zeta_i = \zeta_{i-1}.$$

Therefore, the situation is similar to the one when intervals with numbers greater than i are shifted by unity, which means that lognormality is preserved beyond the considered time interval.

The fact that the water table in lakes is subject to sudden decrease in size is deserving a separate analysis. Given that this may occur irrespective of the proximity of other lakes, the ice plugs tapering out and reappearing in the ground channels are most likely to be the explanation of the water table instabilities [*Makarycheva et al., 2014*]. In such cases, the water table does not reach the rims of the thaw depression, hence, the process of thermal abrasion of the boards is arrested. In other cases, this is attributable to variations in precipitation, during which the lake expansion is supposed to be ceased.

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The analysis has shown that the situation described above, in a minor time interval is analogous to the episodes of ceased growth of lakes. A break in the lake growth having no effect on the lognormal distribution, it can be characteristic of the distribution proportionality coefficient ξ_j in (2), consisting in the fact that some coefficients for some non-zero probability are assumed equal to zero. Provided that we did not impose restrictions on these coefficients distribution and their independence is preserved, this means that the entire procedure of the analysis remains valid.

Empirical testing of the results

The theoretical inference about the lognormal size distribution at any time should be preserved (self-preserving size distributions), and therefore the distribution of the thaw lakes areas, which has been tested at several sites within the permafrost zone, confined to different physiographical, geological and permafrost conditions (Fig. 2) and located in Western and Eastern Siberia, Alaska, Canada. A brief characterization of the areas is given in Table 1. The testing was carried out by digitizing the lakes on the basis of satellite imagery and correlation of the samples (areal extent) with theoretical lognormal distribution using Pearson criterion from the widely applicable methodology [*Cramer, 1970*]. The calculations were performed using "Statistics" software with significance levels of 0.95 and 0.99. The distribution parameters – logarithmic mean and logarithm standard – were estimated by the sampling. The calculated criterion value was compared to the critical value. The latter represents the number of degrees of freedom, depending on the difference between the number of intervals into which the sampling is divided by the program, when testing, and the number of free parameters is estimated in the sample and from a certain confidence level (0.95 or 0.99).

In a number of sample areas, thaw lakes had a simple circular shape. The results analysis has shown that in the overwhelming majority of cases the "lognormal" hypothesis has not been ruled out (Fig. 3, Table 2).

It is much more common that the areas where lakes besides being of circular shape might also have complex outlines. The five investigated areas are located in the West Siberian Plain, within the Pyakupur, Valoktayagun and Vyngapur rivers basins, where from 78 to 214 lakes have been digitized.

The study conducted in the areas has shown good agreement of theoretical and experimental data, as the hypothesis is not rejected with 0.99 confidence level in any of the areas. The study has also ascertained the fact that the interannual variations in the conditions of lakes growth have no significant impact
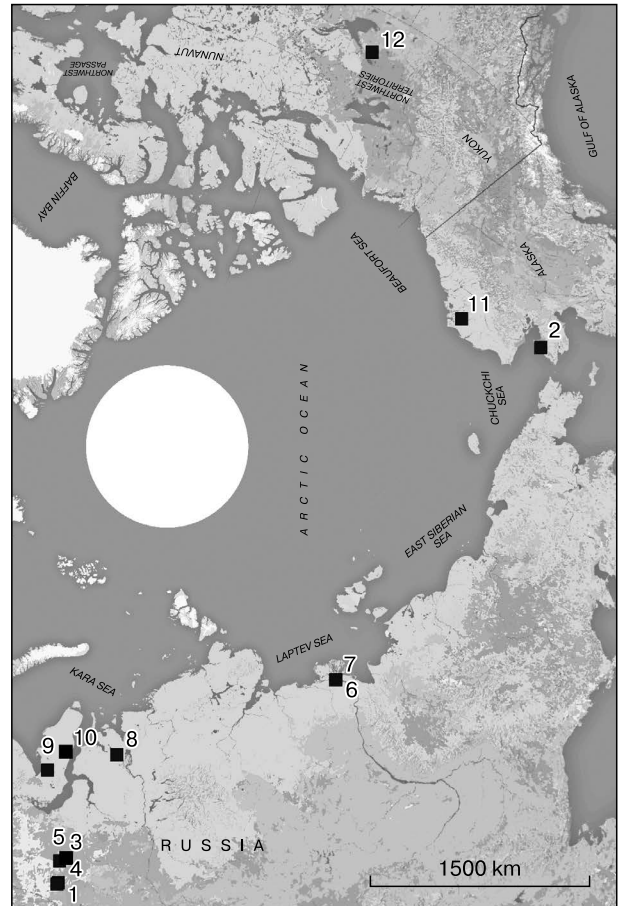

Fig. 2. Schematic representation of the sample areas location:

1 – Valoktayagun-1; 2 – Alaska-1; 3 – Pyakupur; 4 – Valoktayagun-2; 5 – Vyngapur; 6 – Lena-1; 7 – Lena-2; 8 – Gydan; 9 – Yamal-1; 10 – Yamal-2; 11 – Alaska-2; 12 – Canada.

on the compliance with the lognormal distribution. To this end, the satellite images of three sample areas (Valoktayagun-1, Valoktayagun-2, Vyngapur) and four images of the Pyakupur area were examined. In all the cases the samples proved consistent with lognormal distribution, and the data fragment is shown in Table 3.

Similarly, the correctness of the inference on the Poisson distribution of the lakes number in randomly selected sites was also verified. To that effect, an area with digitized lakes was selected in the "Vectorizator" program, and using a special software module we counted the number of lakes centers fallen into the randomly selected (from the random-number generator) site with constant size (in this case, circle). Several series of tests were carried out in each site with their extent changing from series to series. The analysis of the results also shows that in most cases at a

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Table 1. **Characteristic features of sample sites**

Name	Location and description of sample area	Mean annual temperature of air/ rocks, °C	Permafrost thickness, m	Mean annual precipitations, mm
Valoktayagun-1 Valoktayagun-2	**Middle Ob lowland, in the vicinity of the Valoktayagun Rv.** The flat watershed surface area is composed of sandy fluvioglacial deposits (with iciness up to 40 %), overlain by peatland (with ice content up to 90 %). Sparse insular permafrost (perennially frozen rocks, PFR) distribution. The areas with PFR from the surface have a two-layered cross-section of permafrost. The top layer thickness is up to 5 m, relict permafrost strata occur at a depth of 100 m	−4.4/−0.1	up to 5	600
Alaska-1	**Alaska, Serpentine Rv. Delta, Seward Peninsula.** Alluvial-depositional surface is composed of a surface silty sand loams and sands (up to 3–8 m), underlain by sand and gravel stratum, the deposits are peaty. Discontinuous permafrost distribution	−6/−3	5–60	370–450
Pyakupur Vyngapur	**Middle Ob lowland, Pyakupur Rv., Vyngapur Rv.** Flat watershed surface area is composed of lacustrine-alluvial variously grained sands, overlain by biogenic deposits. Discontinuous permafrost distribution	−7/−2	50	410–460
Lena-1 Lena-2	**Lena Rv. Delta area, Chai-Tumus village.** Third above-floodplain terrace composed of icy sandy loam and loam strata, comprising multiple and vertically elongated bodies of ice wedges (ice complex) vertical extent. Continuous permafrost; iciness: 50–60 %	−13/−11	500–700	200–250
Gydan	**Gydan Peninsula.** Coastal-marine plain composed of fluvioglacial laminated fine silty sands. Continuous permafrost. Characteristic is very wide-spread occurrence of epicryogenic permafrost; syncryogenick rocks more than 10 m thick are observable in the cross-sections of lagoon-marine terraces, floodplains and laidas	−11/−9	300–400	350
Yamal-1	**Yamal Peninsula, Turmayakha Rv.** Third lagoon-marine terrace is composed of fine-grained silty sands with rare interlayers of clay loam. Continuous permafrost. Frozen rocks are characterized by almost continuous distribution from the surface and monolithic vertical occurrence; unfrozen rocks are found under the channels of large rivers and lakes	−9/−6	200–280	400
Yamal-2	**Yamal Peninsula, Ust-Yuribey village**. Third marine terrace, flat, swampy, composed of sandy or sand loam deposits. Continuous permafrost. Permafrost is characterized by almost continuous distribution from the surface and vertical monolithic build-up; taliks may occur under the channels of high-water rivers and large lakes	−8/−3	150–250	300–400
Alaska-2	**Alaska, Meade River valley.** Flat plain composed of peaty clay loam and homogeneous fine-grained sands with high ice content of fluvioglacial origin. Continuous permafrost	−12/−6.5	200–300	175
Canada	**Canada, Great Slave Lake.** Lowland plain composed of marine clays, ridgy and palsa bogs with peat thickness up to 3 m are widely spread. Discontinuous permafrost, iciness: 10 %	−5/−2	up to 30	340

significance level of 0.99 (63 out of 66 samples) the Poisson distribution hypothesis is not rejected (Fig. 4).

The satellite imagery timing does not count, either, cf. the data fragment shown in Table 4.

Thus, the results of the testing done prove to be in good agreement with the actual observations of distribution of lake areas (and the number of lakes in randomly selected area) with theoretical assumptions yielded by the model analysis.
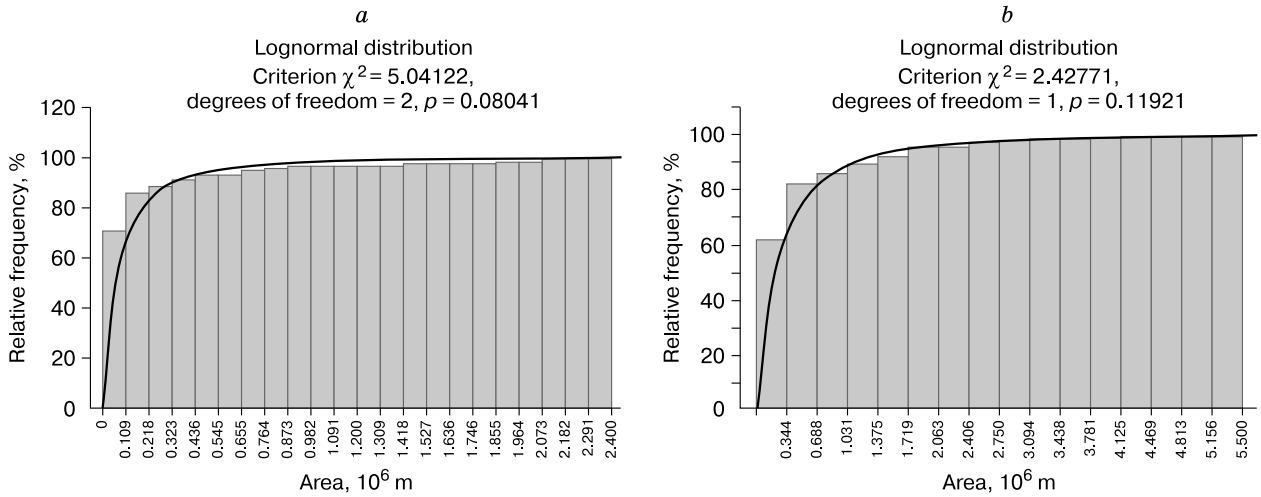
Fig. 3. Illustration of theoretical lognormal distribution (line) consistency with empirical (histogram) distributions of thaw lakes areas:

a – for Lena-2 sample area (Eastern Siberia); *b* – for Vyngapur sample area (Western Siberia).

T a b l e 2. **Comparison of empirical and theoretical thaw lakes areas distributions according to Pearson criterion (χ^2) for the areas with the predominance of circular-shaped lakes**

Sample area	Criterion χ^2	Criterion χ^2 at the 0.95 (0.99) signifi-cance level	ln *s*	*D*
Lena-1	29.09	28.87 **(34.80)**	4.39	1.25
Lena-2	11.10	**12.59 (16.81)**	4.25	1.24
Gydan	7.93	**11.07 (15.09)**	4.35	1.24
Canada	2.20	**5.99 (9.21)**	5.51	2.02

T a b l e 3. **Comparison of empirical and theoretical thaw lakes areas distributions according to Pearson criterion (χ^2) allowing for different timing of surveys (fragment)**

Sample area	Year of survey	Criterion χ^2	Criterion χ^2 at the 0.95 (0.99) signifi-cance level	ln *s*	*D*
Pyakupur	1973	19.16	18.31 **(23.21)**	4.36	1.80
Pyakupur	1987	19.13	16.92 **(21.67)**	4.34	1.80
Pyakupur	2001	24.23	21.03 **(26.22)**	4.37	1.87
Pyakupur	2007	18.29	15.51 **(20.09)**	4.33	1.65

N o t e t o T a b l e s 2, 3. ln *s* is logarithmic mean for the lake area; *D* is logarithm variance for the area. In **bold** font are the values, demonstrating consistency with the hypothesis at a significance level of 0.95 and 0.99.
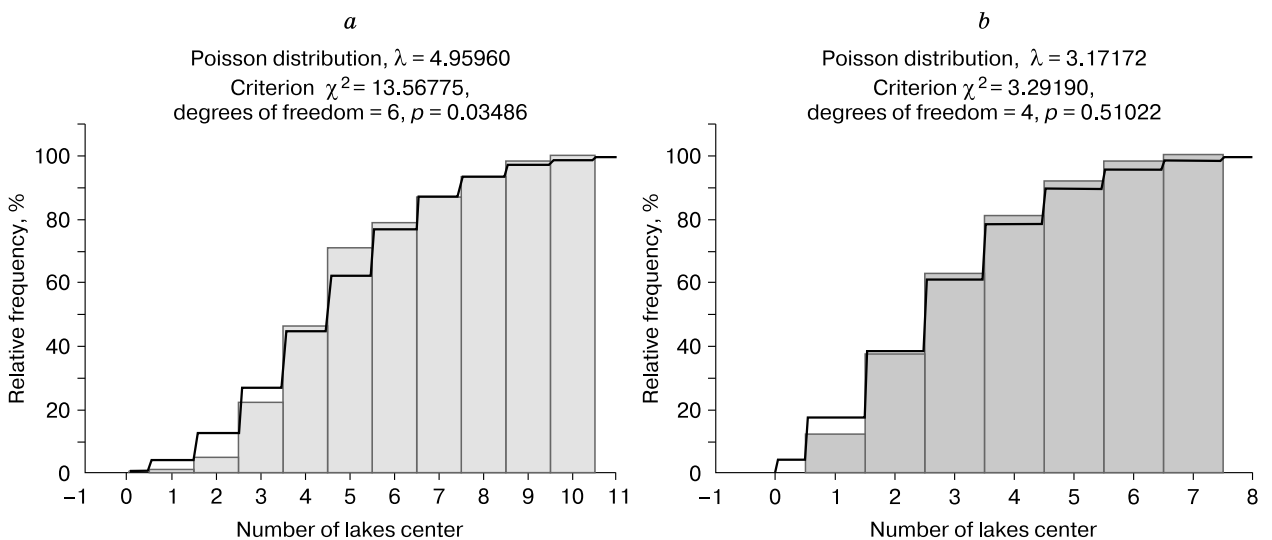

Fig. 4. Examples of consistency of theoretical Poisson distribution (line) with empirical distribution (histogram) of thaw lakes centers in randomly selected areas:

a – for Lena-1 sample area (Eastern Siberia); *b* – for Valoktayagun-1 sample area (Western Siberia).

Table 4. **The fitting criterion value of the empirical and theoretical distributions of the thaw lakes centers (fragment)**

Filename	Circle area, pixel	Average number of lakes in the site	Pearson criterion χ^2	Criterion χ^2 at a significance level of 0.95 (0.99)
Valoktayagun-1 site (1988)	52	3.76	10.74	**11.07 (15.09)**
	45	2.72	6.25	**11.07 (15.09)**
	42	2.61	6.78	**9.49 (13.28)**
	38	2.17	6.34	**9.49 (13.28)**
	35	1.89	2.36	**7.82 (11.34)**
	26	0.84	7.10	5.99 **(9.21)**
	19	0.48	2.86	**3.84 (6.64)**
Valoktayagun-1 site (2001)	87	2.61	9.92	9.49 **(13.28)**
	92	3.00	4.21	**9.49 (13.28)**
	89	2.25	11.64	9.49 **(13.28)**
	76	1.77	5.00	**7.82 (11.34)**
	67	1.28	5.34	**5.99 (9.21)**
	56	1.13	4.19	**5.99 (9.21)**
	48	0.80	0.38	**3.84 (6.64)**
	41	0.51	0.25	**3.84 (6.64)**
Valoktayagun-1 site (2005)	300	4.97	12.63	**12.59 (16.81)**
	286	4.52	1.59	**12.59 (16.81)**
	250	3.58	6.92	**11.07 (15.09)**
	214	2.45	4.41	**9.49 (13.28)**
	206	1.90	2.96	**7.82 (11.34)**
	175	1.69	4.73	**7.82 (11.34)**
	136	0.95	4.04	**5.99 (9.21)**
	119	0.87	4.12	**5.99 (9.21)**
	102	0.64	0.79	**3.84 (6.64)**
	87	0.54	1.65	**3.84 (6.64)**

N o t e. In **bold** font are the values, demonstrating consistency with the hypothesis at a significance level of 0.95 and 0.99.

CONCLUSIONS

1. Modeling results have been obtained for changes in morphological structure of thermokarst lake plains, taking into account climatic changes in homogeneous conditions, in case of isometric lakes.

2. It has been analytically shown that under changing climatic conditions, thaw lakes areas distribution should obey lognormal distribution in every instant, as well as their size distribution should submit to Poisson law.

3. It has been analytically shown that the lognormal distribution of thaw lakes areas and the Poisson distribution of lakes sizes remain valid for different physiographic and permafrost conditions.

4. The empirical research materials for the areas located in different physiographical, geological and permafrost conditions, have proven more than 95 % consistent with the theoretical calculations.

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