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THE EXPERIENCE OF MEDIUM-TERM FORECAST OF THE SEA ICE EXTENT IN THE NORTHERN HEMISPHERE ON THE BASIS OF CALCULATED INCOMING SOLAR RADIATION AND NEURONET MODELING

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Medium-term forecast of the sea ice extent has been carried out by determining of the relationship between incoming solar radiation and the sea ice extent in the Northern Hemisphere. Different methods of statistic and neuronic modeling have been used. The forecast shows that the chief factor determining the variation in the maximum and minimum sea ice extent in the medium-term scale is the variability of solar radiation arriving at the top of the atmosphere. Evaluation of the medium-term forecasts of the sea ice extent demonstrates effectiveness of using the averaged results of the regression analysis and of neural network modeling.

Sea ice, extent change, solar radiation, statistical methods, neural networks

INTRODUCTION

The mean annual area of the sea ice extent in the world ocean is 26 million km^2, or approximately 7 % of its area. In the Northern Hemisphere, the land ice accounts for only 20 % of the total area of the ice cover, while the remaining 80 % are covered by sea ice [*Frolov and Gavrilo, 1997*]. On average, the area of the sea ice extent in Arctic throughout the period of satellite observations (1979–2006) was about 15 million km^2 in February–March and 4.5–5 million km^2 in September [*Fetterer and Knowles, 2004*; http://nsidc.org]. In modern time, reduction in the sea ice extent of multi-year and seasonal sea ice in Arctic related to climate change is observed [*Meier et al., 2007; Wang and Overland, 2009; Ikeda, 2012; IPCC, 2013*]. However, the causes of the long-lasting climate change and the resulting reduction of the sea ice extent in the Northern Hemisphere have not been explicitly determined [*Kondratyev, 1987; Ishibuchi and Tanaka, 1993; Monin and Shishkov, 2000; Badera et al., 2011*]. The unascertained causes of changes in the climatic conditions determining the sea ice extent create a problem in forecasting the sea ice extent of the Arctic seas. The trend of changes in the carbon dioxide content in the atmosphere is an important basis for prognostic solutions used in physical and mathematical models. However, the actual forecasts of changes in the content of CO_2 in the atmosphere do not seem well-grounded but rather conditional; therefore, the forecasts of climatic conditions and of the sea ice extent made on this insecure basis are largely assumptive. At the same time, connection between changes in the sea ice extent and the incoming solar radiation arriving at the top of the atmosphere (TA) has been determined. The values of the insolation have been calculated till 2050 [*Fedorov, 2015a, 2016*]. The objective of this study is to evaluate the possibilities of medium-term forecasting of changes in the sea ice extent on the basis of the calculated insolation values, using statistic methods and neuronic modeling. Forecasting changes in the sea ice extent is an important task due to development of Arctic shipping and oil and gas production in offshore Arctic.

Background insolation data

The values of the incoming solar radiation arriving at the TA were earlier calculated on the basis of astronomic ephemerides (http://ssd.jpl.nasa.gov) by the method developed by V.M. Fedorov and A.A. Kostin [*Fedorov, 2015b,c, 2016*]. The values of solar radiation arriving at the terrestrial ellipsoid in the tropical years, half-years, and seasons were calculated for different latitudinal zones (5°) of the terrestrial ellipsoid in the interval from 1850 to 2050. Based on the computation results for the period of 1850–2050, a database was formed for solar radiation arriving at the TA to the latitudinal zones of the Earth with time increment of 1/12 of the tropical year (http://solar-climate.com). Changes in the solar activity were not taken into consideration.

Data analysis and estimations based on statistical methods

In this study, the obtained values of solar radiation arriving at the TA were compared to the data of satellite survey of the sea ice extent (from 1979 to 2013) in the Northern Hemisphere of the Earth [*Fetterer and Knowles, 2004*; http://nsidc.org]. The authors analyzed two parameters of the sea ice extent of the Northern Hemisphere: the maximum and minimum values of the sea ice extent for many years' periods. Close connection was found between many years' behavior of the sea ice extent and the incoming solar radiation arriving at the TA in the Northern Hemisphere in the summer half-year, as well as the incoming solar radiation arriving at the TA, considering its accumulation. Accumulation of solar radiation was calculated by the authors from the year of the beginning of observations (1979) by sequential addition of the annual values (into the future) and sequential subtraction (into the past). The correlation coefficient between the accumulated solar radiation and the maximum and minimum sea ice extent was –0.835 and –0.852, respectively.

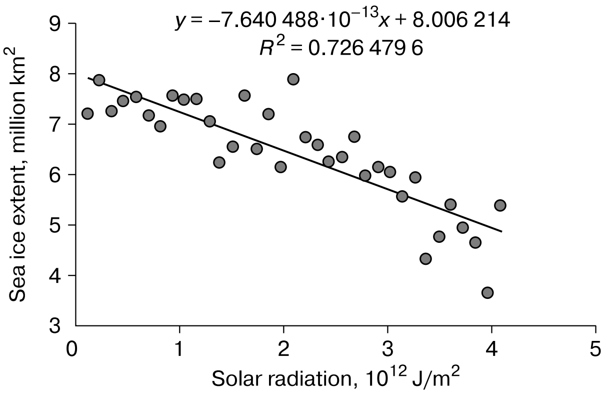

Fig. 1. Linear dependence of the minimum value of sea ice extent on incoming solar radiation arriving at TA in the Northern Hemisphere in the summer half-year.

The linear (Fig. 1) and polynomial forms of connection between accumulated solar radiation and the sea ice extent in the Northern Hemisphere were investigated. The correlation coefficient between the actual and computed values of the maximum sea ice extent was found to be equal to 0.835. The mean annual variance was 0.23 million km^2, which is 1.45 % from the mean annual value of the maximum sea ice extent for the period from 1979 to 2013, or 79.1 % from the mean module of the multi-decadal variance of the maximum sea ice extent (the actual data). The correlation coefficient between the actual and computed values of the minimum sea ice extent was equal to 0.852 (Fig. 2). The mean annual (by module) value of the variance was 0.43 million km^2, i.e., 6.72 % of the mean annual value of the minimum sea ice extent for the period of 1979–2013, or 72.6 % of the mean annual (by module) value of the multi-decadal variance (less than natural climatic variance).

Based on the regression equations obtained, the maximum and minimum sea ice extent values were computed from the period from 1850 to 2050 (Fig. 3). For the maximum sea ice extent, the value computed for the year of 1850 was found to be equal to 21.37 million km^2, and for the year of 2050, the value was 13.33 million km^2. For the year of 1850, the value of the minimum sea ice extent equal to 9.31 million km^2 was obtained, and for the year of 2050, the value was found to be 1.60 million km^2.

Comparison of the results of statistical forecasting to the data provided by physical and mathematical models

There are physical and mathematical models of the total atmospheric and oceanic circulation, on the basis of which the values of the sea ice extent have been calculated for the Northern Hemisphere till 2090 [*Badera et al., 2011; IPCC, 2013; Liua et al.,*

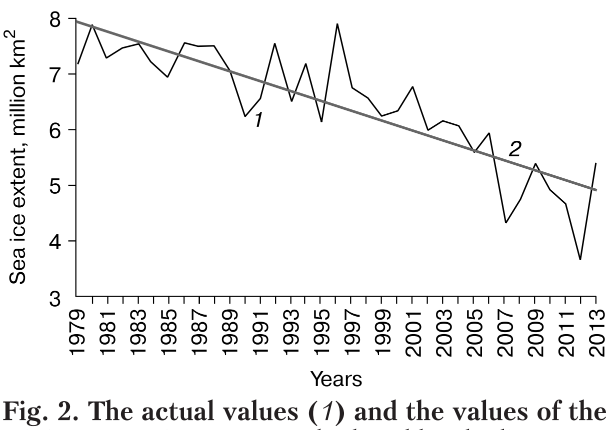

Fig. 2. The actual values (*1*) and the values of the minimum sea ice extent calculated by the linear regression equation (*2*) in the Northern Hemisphere.

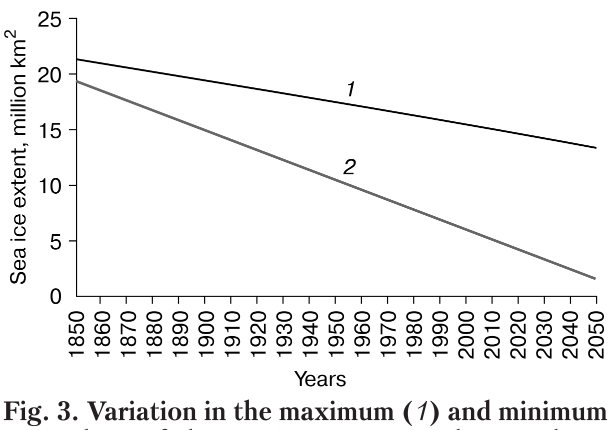

Fig. 3. Variation in the maximum (*1*) and minimum (*2*) values of the sea ice extent in the Northern Hemisphere in the period from 1850 to 2050.

2013; Cassano et al., 2014]. Table 1 shows the mean annual maximum and minimum values of the sea ice extent calculated for the period of 2011–2030 using different physical and mathematical models.

The mean annual values of the sea ice extent obtained by the authors on the basis of a linear regression equation are for the same time intervals 14.52 and 4.23 million km^2, accordingly (the maximum and minimum values of the sea ice extent, accordingly) and are closest to the values calculated by the model of the National Center for Atmospheric Research, USA (the maximum values of the sea ice extent), and the Canadian Centre for Climate Modeling and Analysis (the minimum values of the sea ice extent). It is to be noted that in the calculations based on the regression equations only one factor, the incoming solar radiation arriving at the TA (considering its accumulation), was considered.

As the correlation analysis allows evaluation of the connection between values and processes but does not allow the form of connection between two variables to be determined, we tested application of neural modeling to medium-term estimation of the changes in the sea ice extent.

Table 1. **Calculated area of sea ice extent in the Northern Hemisphere [*Brander et al., 2005*]**

Model	Area, million km^2	
	max (March)	min (September)
CGCM2	15.14	3.33
CSM_1.4	15	7
ECHAM4/0PYC3	15.62	6.03
GFDL-R30_c	15.60	5.91
HadCM3	15.33	6.22

Note. CGCM2 – Canadian Centre for Climate Modeling and Analysis, Canada, CSM_1.4 – National Center for Atmospheric Research, USA, ECHAM4/0PYC3 – Max-Planck Institute for Meteorology, Germany, GFDL-R30_c – Geophysical Fluid Dynamics Laboratory, USA, HadCM3 – Hadley Centre for Climate Prediction and Research, Great Britain.

Comparison of the possibilities of statistical and neural networks methods in investigating time series

Studying the behavior of the sea ice extent generates numerical sequences (naturally associated with certain moments of time) called time series. A time series is statistical data on the parameters of the process under study collected at different moments of time (in the given case, a process of connection between two time series is investigated – the incoming solar radiation arriving at the TA and the sea ice extent in the Northern Hemisphere). Each unit of the statistical material is named measurement or record. In a time series, time of measurement should be indicated for each record. A time series essentially differs from a simple sample of data, as in this analysis interaction between the measurements and time is taken into account, not only statistical variability and statistical characteristics of the sample [*Bogolyubov et al., 2013*].

There are currently a multitude of models solving the forecasting problem. However, the existing methods hardly consider all the factors which can influence the value estimated and precisely determine the dependence between them. This affects the quality of forecasting. The statistical methods are the most widespread methods of analyzing the time series [*Bogolyubov et al., 2013*]. The major challenges of forecasting time series in using statistical methods are the following:

– the absence of operative evaluation of the dependence between the input parameters and the estimated value. For example, the authors have found association between the incoming solar radiation arriving at the TA and the behavior of the sea ice extent. However, the type of this association cannot be unambiguously determined. As a result, given the high correlation degree of the series, the computation values computed by the linear regression equation differ from those obtained in the calculations based on the polynomial regression equation;

– finding the attributes most affecting the forecast value of the attributes and determining the time interval in the past, in which the given variables exert significant influence on the estimated value in the future;

– determining the dependence between the variables found and the estimated value;

– a requirement for high qualifications of the researcher using complex statistical methods.

As a model of complex multi-dimensional nonlinear regression, a neural network exceeds the above methods for its precision degree and has a number of advantages [*Ishibuchi and Tanaka, 1993; Gorban, 1998; Tsaregorodtsev, 2015*]. Neural modeling creates the following possibilities:

– ensuring works with non-information noise input signals, as the neural network can determine their unsuitability for solving the problem and explicitly reject them;

– ensuring works with information of different types: continual and discreet, qualitative and quantitative, which is challenging for statistical methods;

– a neural network places less demands on the qualifications of the researcher using it than complex statistical models capable of providing similar results;

– having originally set the synaptic weights in a neural network, one can retrieve and test the assumed statistical models and improve them by “training” the network [*Wasserman, 1992; Tsaregorodtsev, 2015*].

Considering certain limitations of the correlation analysis related to the fact that it reflects only the linear dependence of the values, not their functional connections, the authors also used parallel calculations using neural network modeling. To forecast time series, a software program developed by O.E. Bukharov on the basis of Nvidia CUDA (Compute Unified Device Architecture) architecture in the Moscow Institute of Electronics and Mathematics of the Higher School of Economics was used. The program was chosen due to its novelty, quality and accessibility, as well as due to the developer's participation in the study conducted. This neural network, which is a three-layered perceptron, unites a genetic algorithm and interval networks [*Bogolyubov et al., 2013; Bukharov and Bogolyubov, 2015*].

Genetic algorithms, the algorithms for solving complex non-formalized problems, are applied to very large-sized problems and in the absence of regular basic data. These algorithms imitate Darwin's evolution theory, searching for solution by sequential improvement of the sets of potential solutions. Improvement of each subsequent set takes place due to crossing and mutations of the best representatives of the previous set of solutions. An interval neural network is a system of interconnected and interacting interval neurons, having the input and output values set as intervals (i.e., not one value but a continual set of values in the range between a pair of values setting the interval limits). Multi-layered networks are formed by cascades of layers, with the output of one layer being the input for another layer. Generally speaking, a multilayered perceptron is one of the most popular neural network models [*Wasserman, 1992*]. Each neuron of the layer receives for its input a sum of weighed outputs of the neurons of the previous layer. At the output, each neuron has a value of its activation function from the output. To train the multi-layered perceptron, the algorithm of error backpropagation is applied, based on the method of gradient descent.

Analysis and forecasting based on neural network modeling

As input data, the same results of the satellite observations of the sea ice extent and the previously calculated insolation data [*Fetterer and Knowles, 2004;* http://nsidc.org; http://www.solar-climate.com] were used.

As input data, the system was offered arrays of the following annual parameters from 1979 to 2013:

– interval values of the sea ice extent (from minimum values in September to maximum values in March);

Table 2. **Estimated change of the sea ice extent in the Northern Hemisphere**

Year	Sea ice extent, million km^2	
	min	max
2014	5.40	14.48
2015	5.54	14.20
2016	5.60	14.30

– incoming solar radiation arriving at TA in the summer half-year in the Northern Hemisphere;

– incoming solar radiation arriving at TA in the summer half-year in the Northern Hemisphere considering its accumulation;

– the difference between the solar radiation values arriving at TA in the equatorial and polar regions of the Northern Hemisphere for the year and the summer half-year (considering accumulation of solar radiation and without it);

– the ordinal number of the year.

Resulting from application of the system to solving the problem of forecasting the sea ice extent, the networks using the incoming solar radiation arriving at TA as one of the input parameters proved to be found in the pool of the quality information systems. This suggests that this parameter has been consistently selected as a factor exerting essential influence on the forecast value. Comparing the forecasts obtained by the system for the years with the previously known parameters to the actual values of the sea ice extent, one can see the quality of the forecast and sufficiency of the historical knowledge (needed for training the system) of the sea ice extent and of the knowledge of the incoming solar radiation arriving at the TA. The standard mean-root square error of forecasting for the forecasting depth of 1 year is 0.002 82, that for the forecasting depth of 2 years is 0.004 82, and that for the forecasting depth of 3 years is 0.005 62. These results were obtained in training the system on historical data on the sea ice extent and the incoming solar radiation arriving at TA from 1979 to 2008. The test forecasts and evaluation of the forecasting quality were conducted on the historical data of 2009–2013.

To calculate the standard mean-root square error of forecasting, the formula was used,

$$Error = \left(\frac{\hat{x} - x}{\Delta x}\right)^2,$$

where $\hat{x}$ is the predicted value of the variable; x is the actual value of the variable; $\Delta x = x_{\max} - x_{\min}$.

Considering the obtained evaluations of the test results, changes in the sea ice extent were estimated for the period of 2014–2016 without actual data on the ice situation (Table 2).

Comparison of the forecasting results on the basis of statistical data and neural network modeling

The values of the sea ice extent for the period from 2014 to 2016 calculated with neural networks were compared to the forecast values calculated by the linear and polynomial regression equations (Table 3). It follows from comparison of the forecast values of the sea ice extent that:

1. The results of forecasting the minimum sea ice extent at neural network modeling exceed the respec-

Table 3. **Estimated values of sea ice extent based on analysis of satellite data, million km^2**

Year	Values calculated with the regression equation: linear equation	Values calculated with the regression equation: 2nd-degree polynomial	Values calculated with neural networks	Difference of results by neural networks and linear regression equation	Difference of results by neural networks and polynomial regression equation
			Maximum sea ice extent		
2014	14.78	14.83	14.48	–0.30	–0.35
2015	14.74	14.78	14.20	–0.54	–0.58
2016	14.70	14.75	14.30	–0.40	–0.45
Mean value	14.74	14.79	14.33	–0.41	–0.46
			Minimum sea ice extent		
2014	4.80	4.15	5.40	0.60	1.25
2015	4.71	3.95	5.54	0.83	1.59
2016	4.63	3.75	5.60	0.97	1.85
Mean value	4.71	3.95	5.51	0.80	1.56

tive values calculated by regression equations. Reversely, when forecasting the maximum sea ice extent, the values calculated with the neural networks were less than those obtained from the regression equations.

2. In calculating the maximum sea ice extent, the mean variance between the values calculated by neural networks and by linear and polynomial (fuzzy) regression equations are close (–0.41 and –0.46). In forecasting the minimum sea ice extent, the mean variance between the values calculated with neural networks and with the polynomial regression equation nearly doubled the mean variance between the values obtained with neural networks and with the linear regression equation.

3. The absolute values of the variance in forecasting the minimum sea ice extent essentially exceed the variance in the values obtained with neural networks and the values calculated with the regression equations in forecasting the maximum sea ice extent.

The mean variance in forecasting the maximum sea ice extent with neural networks is 2.95 % of the mean value of the sea ice extent, calculated by the regression equations. When the minimum sea ice extent is estimated, this variance increases compared to the results calculated by the linear regression equation to 16.99 %, and compared to the results calculated by the polynomial regression equation – to 39.49 %. This occurs due to both increase of the value of variance and reduction of the absolute values of the sea ice extent. Thus, the values closest to those calculated with neural networks were obtained with the results calculated by a linear regression equation. In forecasting the maximum sea ice extent they are lea than in forecasting the minimum sea ice extent.

To compare the forecast values of the sea ice extent with the actual values, the forecast values were calculated on the basis of more continual time series (1870–2007) по sea ice extent [*Walsh and Chapman, 2001; IPCC, 2013; Fedorov, 2015a*]. The continuity of the time series allowed us to make a forecast for the period from 2008 to 2012 for which satellite data were available (assumed to be actual data). In neural network forecasting the mean standard error for the test series was 0.003 45, the maximum standard error was 0.053 79.

The values of the maximum sea ice extent calculated for the period of 2008–2012 by the linear regression equations and with neural networks were compared to satellite data [*Fetterer and Knowles, 2004;* http://nsidc.org] (Table 4). In this case, the values obtained by means of neural networks exceed

Table 4. **Estimated values of maximum sea ice extent based on reconstruction of maximum sea ice extent [*Walsh and Chapman, 2001*], million km^2**

Year	Values based on satellite data	Values calculated with the regression equation: linear equation	Values calculated with the regression equation: 2nd-degree polynomial	Values calculated with neural networks	Mean values calculated with the linear regression equation and with neural networks	Difference between calculated mean values and actual values
2008	15.22	15.02	15.03	15.39	15.21	0.01
2009	15.14	14.98	14.99	15.59	15.29	0.15
2010	15.11	14.94	14.96	15.36	15.15	0.04
2011	14.58	14.90	14.92	15.32	15.11	0.53
2012	15.24	14.86	14.89	15.51	15.19	0.05
Mean value	15.06	14.94	14.96	15.44	15.19	0.16

Table 5. **Estimated values of minimum sea ice extent based on reconstruction [*Walsh and Chapman, 2001*], million km^2**

Year	Values calculated with the linear regression equation	Values calculated with neural networks	Difference between the results calculated with the linear regression equation and with neural networks	Module of difference of results
2008	8.85	8.82	0.03	0.03
2009	8.82	9.44	–0.62	0.62
2010	8.80	8.86	–0.06	0.06
2011	8.78	8.90	–0.12	0.12
2012	8.76	9.12	–0.36	0.36
Mean value	8.802	9.028	–0.24	0.24

the values of the maximum sea ice extent calculated by the linear and polynomial regression equations. The values calculated with neural networks also exceed all the respective satellite (actual) data in this interval on average by 0.38 million km^2, or 2.50 % of the mean annual value of the maximum sea ice extent in the period of 2008–2012. The mean value of the variance between the values of the maximum sea ice extent calculated by the linear regression equation is 0.25 million km^2, or 1.63 % of the mean annual value of the maximum sea ice extent in the period of 2008–2012. The mean variance between the averaged (by two calculation methods) values of the maximum sea ice extent calculated with neural networks and by the linear regression equation and satellite (actual) data by the module is 0.16 million km^2, or 1.04 % of the mean annual value of the maximum sea ice extent in the period from 2008 to 2012, i.e. in this case variance (by absolute values) decreases 2.4 times.

We have not been able to make a similar comparison with the minimal values of the sea ice extent, as continuous series [*Walsh and Chapman, 2001*; http://arctic.atmos.uiuc.edu] differ much from the satellite data [*Fetterer and Knowles, 2004;* http://nsidc.org] in terms of the minimum sea ice extent. This is likely to be related to differences in their calculations (some are calculated for the end of September, the others are mean monthly values for September). However, concordance of the minimum values of the sea ice extent calculated on the basis of this series with the neural networks and by the linear regression equation is rather high (Table 5).

The mean variation by the module is 4.01 % of the mean for the period of 2008–2012 minimum sea ice extent by the linear regression equation, and 3.91 % of the mean sea ice extent calculated with neural networks. In this case, the difference module (0.24 million km^2) is much less that that obtained by a short series (0.80 million km^2, Table 3). However, these differences are much less than the differences between the mean annual values of the sea ice extent obtained by the calculations based on individual physical and mathematical models (Table 1).

CONCLUSION

1. The experience of forecasting variation in the sea ice extent with various mathematical methods applied (statistical and neural network modeling) shows that incoming solar radiation arriving at the top atmosphere is the most significant factor determining the multi-decadal variations between the maximum and minimum values of the sea ice extent.

2. The use of the averaged results of the regression analysis and of the results of neural network modeling has been shown to be effective for medium-range forecasting of the sea ice extent. Thus, combined application of statistical methods and neural network modeling (statistical and neural network methods ensemble) seems to be promising for medium-range forecasting of the variation in the sea ice extent.

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