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DETERMINATION OF DIELECTRIC PERMITTIVITY FROM DIFFRACTION TRAVELTIME CURVES WITHIN A DIPPING-LAYER MODEL

S.V. Popov

Polar Marine Geosurvey Expedition (PMGE),
24, Pobedy str., Lomonosov, St. Petersburg, 198412, Russia; spopov67@yandex.ru

The main objective of the GPR data processing and interpretation is to select a velocity model for time-to-depth conversion. The modeling results allow to equate the distinguished layers with geological or glaciological objects. The paper describes an algorithm for calculating the diffraction traveltime curves realized within a model of dipping layer medium with an aim to construct appropriate velocity model. The efficiency of the described algorithm is illustrated by the GPR data collected during the austral summer field season of the 60th Russian Antarctic Expedition (2014/15) in the Mirny Station area, East Antarctica.

GPR investigations, traveltime curve of diffracted wave, velocity model of the media

INTRODUCTION

When conducting GPR radar surveys and radio-echo sounding (RES), a seismic interpreter faces essential and sometimes fundamentally important problem of the choice of a velocity model for data conversion from time to depth domain. Depending on the application tasks and the required calculation accuracy, the following three basic approaches are most widely used to that effect: 1) application of tabulated values based on a priori concepts of inherent properties of the propagation media; 2) specific measurements, e.g., multi-offset sounding [*Sheremetiev, 1989; Popov et al., 2001; Popov et al., 2003; Vladov and Starovoytov, 2004; Macheret, 2006; Glazovskii and Macheret, 2014*]; 3) the use of diffraction traveltime curves in case of inhomogeneous media [*Popov, 2002; Glazovskii and Macheret, 2014*]. In view of its simplicity and efficiency, the latter approach has become widely applicable. The method for determining wave propagation velocity in a horizontally layered medium is discussed in [*Forte et al., 2013*]. With this in mind, it is suggested, measuring both time delays, and amplitudes of the signal reflected by target boundaries of the medium, i.e. including reflection amplitudes analysis [*Forte et al., 2013, 2014*]. The use of diffraction traveltime curves applied to time-to-depth velocity conversion is provided for in all geodetic data processing packages known to the author, such as GeoScan32 (LLC Logic Systems), RADAN (Geophysical Survey Systems, Inc.), RadExPro (Deco Geophysics UK) and matGPR (Andreas Tzanis, University of Athens). However, judging by the methodology and User Guides prescribing how to handle them [*GeoScan32..., 2009; Tzanis, 2010; RADAN 7 Manual, 2013; RadExPro..., 2013*], the simulation of a diffraction traveltime curve proceeds from the assumption that the considered medium above the reflector is homogenous and isotropic. The selection of parameters thus results in obtaining a certain *effective* value of kinematic attributes. If layers with different permittivities are located above the reflector (for natural media, they vary over a very wide range: from the first units to hundreds [*Vladov and Starovoytrov, 2004; Macheret, 2006*]), the obtained effective value may fail to be interpreted correctly, leading therefore to misinterpretation of the GPR data. The paper proposes a new algorithm for calculating diffraction traveltime curves and a new technique for its application to determining the dielectric permittivity of a medium.

PROBLEM FORMULATION AND CALCULATING ALGORITHM FOR DIFFRACTION TRAVELTIME CURVE

There is an empirical evidence that that the surveyed subsurface with a length comparable to its thickness, generally, can be studied within the two-dimensional dipping-layer model with flat refractors, which, when applicable to real geological objects, appear to be the opposite, though. For practical applications of the proposed model the digitized boundaries between its media must be therefore approximated by straight lines, in its section.The least squares approach represents an optimal method for such approximations. Let's assume that there are N inclined (in the general case) interfaces on the GPR time section, which determine the configuration of number of layers with known dielectric permittivities ε_i (i is the layer's upper interface). To be specific, we will consider the first (shallowest) interface to correspond to the horizontal surface with the GPR antennas laid on it. Suppose that in the region below the bottom layer there is a reflector at the point with (x_N, y_N) coordinates, generating a diffracted event confidently recorded on the time profile. We assume that both the

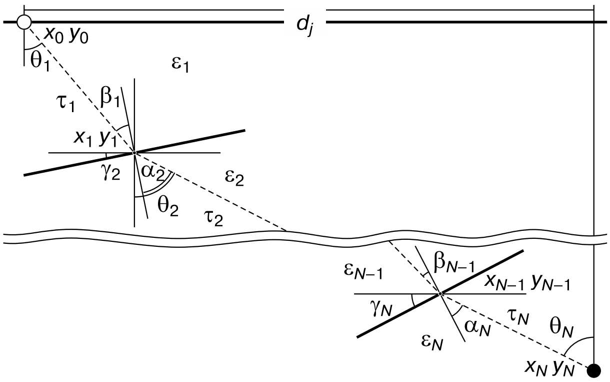


Fig. 1. Schematics illustrating the algorithm for generating a diffraction traveltime curve (see explanations in the text).

reflector and GPR profile line lie on the same vertical plane. Given the overlying layers ε_i are known, it is required to determine permittivity of the ε_N region with the reflector located within its bounds. The model of the surveyed medium is shown in Fig. 1.

We will solve the problem, selecting ε_N by trail-and-error method. To do so, we form rays propagating at some given incidence angle θ_N to the vertical axis from the reflector to the zero boundary. If d_j is the distance along the horizontal axis between the intersection points of the j-th ray with the zero boundary and reflector, while τ_j is the total propagation time of the EM wave along the ray path (Fig. 1), then the set of j-th points (d_j, τ_j) of the N layer represents a theoretical traveltime curve of the diffracted wave produced by the reflector. By varying ε_N, we select the theoretical traveltime curve in such a way that it coincides with the observed one.

The digitized boundaries on the profile intervals intersected by rays with specified θ_N will be approximated by inclined straight lines, once the coefficients have been calculated by the least squares method (Fig. 1). In this case, for each i-th boundary, the coefficient of the linear term is equal to the tangent of its inclination angle to the horizontal axis γ_i. Then we isolate a set of rays generated by the reflector at different angles θ_N, with a unique pair (d_j, τ_j) designated for each ray path, which will enable plotting the traveltime graph.

Consider the j-th ray starting from point (x_N, y_N) of the reflector's position which falls on the interface between the upper medium $N-1$ at point (x_{N-1}, y_{N-1}) with the angle of incidence α_N (Fig. 1). Wherein,

$$\alpha_N = \theta_N - \gamma_N. \tag{1}$$

Since dielectric permittivities at the top and bottom of this boundary are equal to ε_{N-1} and ε_N, respectively, then, according to Snell's law, the angle of refraction β_{N-1} in layer $N-1$ has a value allowing to fulfil the relation

$$\sin\beta_{N-1}\sqrt{\varepsilon_{N-1}} = \sin\alpha_N\sqrt{\varepsilon_N}. \tag{2}$$

Now we analyze $N-1$ layer. Angle $\theta_N - 1$ between the ray and vertical axis is such that

$$\theta_{N-1} = \beta_{N-1} + \gamma_N. \tag{3}$$

To determine the value of angle of the ray incident on the overlying boundary, it is necessary to reduce ratio (1) by one, and to decrement (2) and (3), accordingly. Gradually moving upwards from one boundary to another, and thereby decreasing indices, we will consecutively determine positions of the refraction points on the i-th layer (x_i, y_i), which determines value (x_0, y_0), and consequently, d_j. The total propagation time of EM wave τ_j along the considered j-th ray is written as

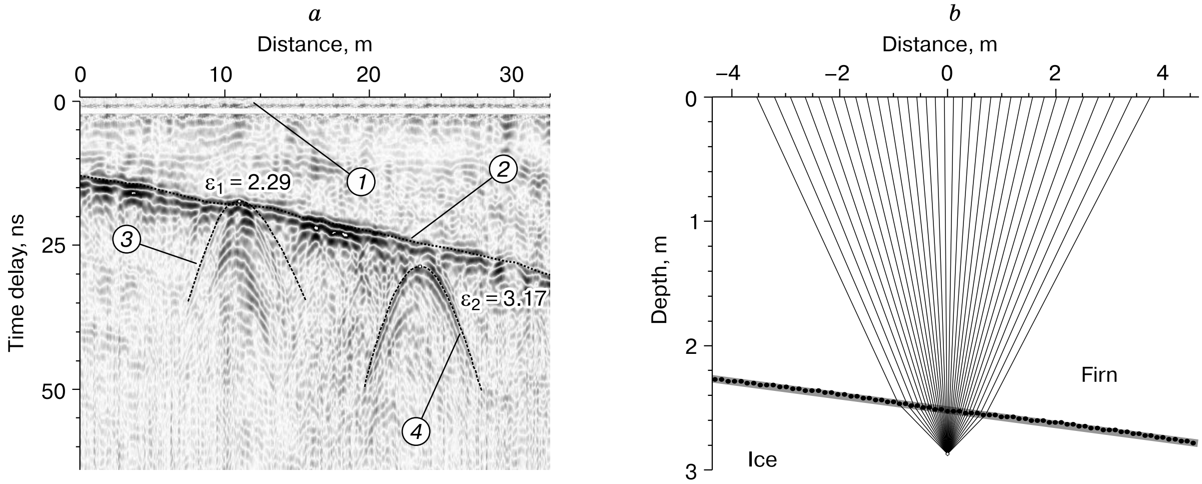
$$\tau_j = \frac{2}{c}\sum_{i=1}^{N}\sqrt{\frac{\left(x_i - x_{i+1}\right)^2 + \left(y_i - y_{i+1}\right)^2}{\varepsilon_i}}, \tag{4}$$

where c is EM waves propagation velocity in vacuum. Visualizing (d_j, τ_j) pair for all j in a handy way we obtain the required *theoretical diffraction traveltime curve* within the dipping-layer model.

MODEL APPLICATION TO THE GPR DATA PROCESSING

As an example of the GPR data processing in glaciological applications, below we analyze a fragment of the time section obtained during the austral summer field season of the 60th Russian Antarctic Expedition (RAE, 2014/15) in the area of Russia's Mirny Observatory (East Antarctica).

The task of the conducted geotechnical surveys consisted in localizing and mapping the near-surface englacial crevasses in order to select the construction site for a new airfield [*Popov et al., 2015*]. With some of the cracks capable of reaching several tens of meters in width, these phenomena have had a bad record at Mirny station [*Pryakhin et al., 2015*], which does provide additional challenges to both daily life of the station and logistic operations. Particularly great difficulties were encountered at the time when Mirny station was made the base camp for sledge-caterpillar traverses in the interior regions of Antarctica [*Popov and Popkov, 2015; Popov, 2015*]. Despite being a great adversary, the crevasses in ice sheets, due to their remarkable length and depth turned out to be excellent reflectors, generating in most cases numerous diffracted waves with centers located at different depths [*Glazovslii and Macheret, 2014; Popov and Eberlein, 2014; Popov and Polyakov, 2016*]. This, in turn, allows to construct adequate representation of velocity structure of the medium.



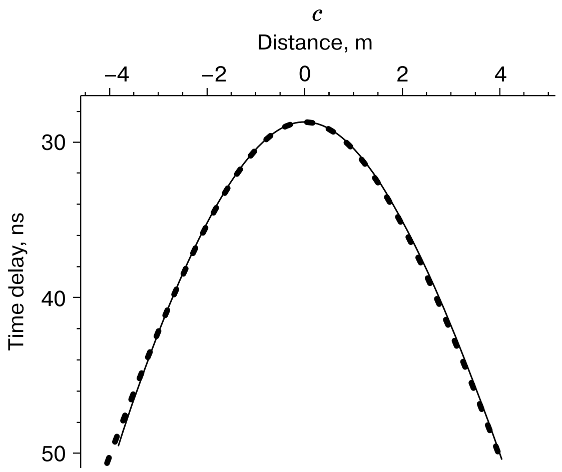


Fig. 2. GPR time-section (*a*), ray paths (*b*) and traveltime curves of diffracted waves (*c*).

a: *1* – reflections from the glacier surface; *2* – the assumed reflection from the interface between firn and glacial ice; *3*, *4* – diffracted waves. ε_1, ε_2 – the dielectric permittivities of the media calculated from diffraction traveltime curves. *b*: black dots: digitized target boundary; the gray line is its least-squares approximation; the thin lines indicate rays produced from the diffraction point. *c*: the dashed line is s traveltime curve formed within single-layer medium; a solid line is a traveltime curve generated within the dipping-layer medium.

Fig. 2, *a* shows a fragment of the GPR time section along one of the geophysical survey routes [*Popov et al., 2015*]. Reflection event *1* is associated with the glacier surface. Its low intensity stems from the result of the data processing procedure for coherent-noise suppression. Contrast reflection event *2* may indicate an interface between a snow-firn layer and glacial ice.

Two diffracted events (*3* and *4*) are distinctly traced in the time section (Fig. 2, *a*). The reflector producing diffracted event *3* is located at the interface between these media. Therefore, its gather allows to determine velocity parameters of the upper medium within the limits of a homogeneous medium model. According to calculations, value $\varepsilon_1 = 2.29$ corresponds to the firn layer [*Macheret, 2006*]. Alternatively, the calculation for the traveltime curve of event *4* diffracted from the reflector positioned below target boundary *2* yields a value of 2.39 within the homogeneous medium model, which is also interpreted to be the firn layer [*Macheret, 2006*]. Note that the previous value does not differ greatly. Considering the above two-layer model, the obtained value $\varepsilon_1 = 2.29$ is used for dielectric permittivity of the upper layer. The theoretical traveltime graph, calculated from relation (4), matches best with the one recorded at $\varepsilon_2 = 3.17$, which is called "classical value" of dielectric constant for glacial ice [*Macheret, 2006*]. The ray paths are shown in Fig. 2, *b*. A comparison of the two traveltime curves constructed within the framework of these models is shown in Fig. 2, *c*. Thus, the transition from the homogeneous medium model to a dipping-layer model has dramatically changed the interpretation of the GPR time-section. According to the former, the entire section is composed of a fairly uniform layer of firn, while the latter, more correct, model shows that the firn layer extent is delimited by boundary *2*, with the glacier ice being subjacent to it.

DISCUSSION

The example discussed in the previous section has shown that the dipping-layer model application to analysis of diffraction traveltime curves allows to define far more precisely the velocity characteristics of the concerned propagation media. At first glance it may seem that introduction of this approach is capable to replace some of the dedicated surveys, specifically, the multi-offset sounding technique. However, it appears very much to be more a tradeoff, than the truth. The proposed method is not perfectly free of shortcomings. First thing, the survey line is assumed to traverse the reflector, which is not always the case, though. If the reflector position stands off the survey line, then the analysis of the equation for the traveltime curve shows no signs of changes in terms of determining the velocity parameters of a single-layer medium. The out-of- plane (lateral) diffraction will only increase the distance to the reflector [*Popov and Kashkevich, 2015*].

Thus, the traveltime curve configuration, as previously, will allow to correctly determine dielectric parameters of the medium; the depth of the reflector, however, will be thereby overestimated. In case of the target medium containing more than one layer, accurate determination of the velocity parameters from the traveltime curves is all but impossible, as rays corresponding to the apex position of the diffracted wave are not vertical, and the number of unknowns exceeds the number of equations. If the diffracted wave is confidently traced and can principally be computed, what most makes sense is to possibly traverse the survey line at the diffraction point.

Another deficiency is that not all contemporary GPRs have integrated antennas. In practice, this means that if the depth of the reflector is comparable to the distance between the antennas (base), then it is necessary to use the relations for *two-position soundings* to obtain higher accuracy. But this appears adequate only for relatively shallow depths and large bases [*Popov and Kashkevich, 2015*].

One more disadvantage consists in the accuracy of determining the velocity characteristics which fully depends on the accuracy of the georeferencing of sounding points, and as to what extent the route is straightforward. Although these parameters appear fundamental in conducting specialized measurements (in particular, multi-offset sounding), the survey works will receive less attention, whether from industry or science, and are sacrificed to other, more demanding, tasks and to time-saving solutions. One of the ways out is to calculate as many traveltime curves as possible to accumulate data for the statistics.

And finally, the last argument. The dipping boundaries are determined from the digitized time section of the GPR profile. However, it has been known from [*Popov and Luchininov, 2001; Vladov and Starovoytov, 2004*] that their configurations are not real: one needs to perform time-to-depth conversion using time-migration velocity in order to locate their true position, which also requires constructing a velocity model of the medium. The situation is favored by the fact that with small angles of inclination, configuration of the target boundaries on the time sections generally coincides with the true one. Given that the angles of inclination reach essentially big values, we have to face the fact that the resulting profile will differ from the real one. Despite the discussed shortcomings of the proposed approach, it nevertheless allows, in the absence of a priori data and results of dedicated works, to obtain fairly valid velocity characteristics of the identified layers.

CONCLUSION

The formulated in this paper problem of determining velocity parameters of the dipping layer medium is topical in contemporary geophysics. The practical experience shows that the existing understanding about the geological structure of the study area does not sufficiently provide for correct interpretation of GPR acquisitions. Thus, the tabulated data on permittivity of rocks, among other things, attest to their high sensitivity to moisture content. This is equally important in glacier studies.

According to the ice core data, their upper portion is composed of snow-firn stratum whose heterogeneity increases with depth [*Ekaykin et al., 2014; Vladimirova et al., 2015*], which in the coastal part of Antarctica, is sometimes intruded by thick layers of glacial ice [*Polyakov et al., 2015*]. Given that there is a direct relationship between density and permittivity [*Robin, 1975*], and that of the snow-firn density varies widely, the occurrence depth of the glacier bed can not be determined accurately without specific applications. The author hopes that the above approach can be helpful in the construction of a valid velocity model for the GPR-surveyed medium.

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