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GUIDELINES FOR DETERMINING TANGENTIAL FORCES OF FROST HEAVING OF SOILS

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A comparative determination of tangential forces of frost heaving depending on the soil freezing temperature regime was carried out using three calculus methods. Their analysis resulted in recommendations (relying on standard laboratory method for measuring specific tangential forces of frost heave of soils) for calculating integral magnitude of the tangential forces of frost swelling of foundations.

Frost heave of soils, tangential force, shear resistance, foundations, regulatory documents

INTRODUCTION

Designing structures for building on soil bases susceptible to frost heaving is associated certain challenges faced in determining the magnitude and variation of forces affecting the structures' foundations during soil freezing. The forces of frost heaving are basically divided into normal stresses, acting in the direction of the temperature gradient (which can be vertical and horizontal), and tangential stresses (shear stresses) acting along the side of foundations.

This paper considers in detail tangential frost heaving stresses with an emphasis on the application of normative documents at the engineering design and construction phases. It should be noted that modern normative documents in the field of construction do not contain a warning about non-compliance with the standard punishable by law, inasmuch as the current technical regulation system in the Russian Federation is based on the principle of voluntary application of standard prescribing documentation (No. 184-FZ as of 27.12.2002 and No. 162-FZ as of 29.06.2015). The technical regulations make allowance for the fact that the previously existing standards often become outdated with introduction of new results of scientific and applied research.

Given that the standards updating procedure takes quite a long time, it is allowed now to use more advanced standards and methods for determining soils characteristics, in the cases when they prove to provide more accurate and reliable input data for the design of engineering structures. Such verification is usually required at the stage of the state expert review of the geotechnical survey materials.

Accordingly, the use of standard requirements applicable to the laboratory determination of specific tangential frost heaving forces (force per unit volume) in soils [*GOST R 56726-2015*] was questioned when calculating the integral tangential frost heaving force in accordance with the Construction Code [*SP (Svod Pravil) 25.13330.2012. Amendment No. 1*], which failed to appreciate the effects of soil temperature distribution during the period of seasonal freezing on the formation of tangential frost heaving force (shear stresses).

This applied research underpinned by scenario-based calculations and field work materials, is set out to optimize the determination of integral tangential frost heaving force which takes into account temperature field of the soil freezing processes. The results obtained are applicable to developing methods for rating of frozen soils in regard to their engineering properties.

BASIC PROVISIONS

The magnitude of tangential frost heaving force is included in the calculus schemes for determination of the bearing capacity of the foundations of structures erected on soils affected both by permafrost and deep seasonal freezing [*SP 25.13330.2012*]. The magnitude of tangential frost heaving forces can be estimated both on site and in the laboratory conditions. The results of field studies taken into account when developing methods for laboratory assessment of tangential frost heaving forces can improve their reliability. The main inferences derived from the field studies, which are found critical for the laboratory tests, are discussed below. Tangential frost heaving forces are implemented through the effects of soil adfreezing with the side surface of the foundation. The pioneering calculus method for structures stability on the frozen ground was proposed by *N.A. Tsytovich* [*1928*].

The first attempts to directly measure tangential frost heaving force were made by N.I. Bykov [*Bykov and Kapterev, 1940*]. The method consisted in measuring the maximum frost heaving forces when testing model piles with static loads. Despite the fact that the method enabled determination of the magnitude of frost heaving forces, it however failed to explicate the soil freezing process dynamics.

The dynamometric method developed by S.S. Vyalov allowed studying the dynamics, i.e. time- and depth-dependent evolution of frost heaving forces [*Vyalov and Egorov, 1958*]. Based on the results obtained, S.S. Vyalov proposed to use the specific tangential frost heaving force corresponding to the their maximum integral magnitude for calculating buckling forces in foundations. This provision was implemented in the building codes.

The equipment used for laboratory determination of shear resistance of frozen soils along the lateral surface of foundations, allowed the foundation model to be punched through the tested soil. N.A. Tsytovich used beams (40 mm in diameter and 120 mm in height) [*Tsytovich, 1973*], while the plates in devices with a two-plane shift were used by V.F. Ermakov [*Guidelines..., 1973*], as a foundation model.

In his methodological work, L.T. Roman showed that pile diameter does affect the magnitude of tangential freezing (and heaving) forces under natural conditions [*Roman, 2002*]. At that, the smaller is the diameter of a pile, the lower is the value of the tangential frost heaving force. This effect however ceases, once the pile diameter reaches 200 mm or more (according to the Fundamental Project Institute data, 140 mm or more). A further increase in the pile diameter has not influenced the measurement results. Using the pile model in the form of a plate by V.F. Ermakov is therefore justified.

B.I. Dalmatov and other researchers [*Dalmatov, 1957; Dubnov, 1967; etc.*] investigated tangential frost heaving forces in laboratory conditions. In his experiment with driving model piles adfrozen into soil with different displacement rate, B.I. Dalmatov obtained an important resulting inference: the magnitude of long-term adfreezing strengths is close to that of tangential frost heaving forces of soils. These results were further developed in [*Dubnov, 1967*], providing a complicated dependence of the variation of the adfreezing effects on the duration of shear stress (Fig. 1).

On *the first intercept* (I) of this curve the shear resistance of soil graded into frozen state along the foundation surface, whose magnitude increased abruptly to the maximum even at the slightest displacements of model pile in soil. This is associated with the appearance of ice-cement bonds in soils, ensuring thereby an increase in the strength of soil adfreezing with the foundation surface.

The second intercept (II) of the curve is characterized by a sharp decrease in shear resistance, which occurs due to the static ice-cement bonds between the soil and the foundation affected by freezing.

The third intercept (III) of the curve displays a decline in the shear resistance as decelerating and gradually approaching its constant long-term value.

The third intercept represents either the stage of stabilized plastic-viscous deformation, creep (after S.S. Vyalov), or the period of tangential stresses achieving the value of long-term resistance to adfreeze strength (after B.I. Dalmatov), or the stage of long-term shear resistance (after S.S. Vyalov and V.O. Orlov) [*Orlov et al., 1977*].

In some cases, when the active layer heaving is involved, the process of continuous displacement of soils along the lateral surface of the foundation can have an intermittent pattern. The process of adfreeze strength recovery after the interrupted movement of pile therefore appears interesting.

Figure 1 shows the effect of pile stoppage on the dynamics of strength of soil adfreezing with the pile material [*Dubnov, 1967*]. As it follows from the experimental data, the interrupted freezing of soil along the foundation model leads to a decrease in the adfreeze strength to a certain constant value. The resumed movement of the foundation model relative to the frozen ground is accompanied by a repetition of all three stages of changes in the adfreeze resistance (with a smaller amplitude). Importantly, the value of ice-cement adhesion of frozen soil with the foundation material is not recovered to its maximum value, despite the durability of stoppage episodes. Therefore, the maximum adfreezing resistance can be used as a design characteristic in the initial period of the active layer heaving, when the soil layer frozen to a depth of 0.3 m becomes steadfastly adfrozen with the lateral surface of the foundation. While further heaving of the active layer leads to the destruction of the initial contact bonds, triggering thereby the frozen layer displacement along the lateral surface of the foundation.

According to the experimental data, the maximum adfreeze resistance can reach large values of the order of 1.0–1.2 MPa (at a temperature of −10 °C, shear rate of 10 mm/day and moisture (water) content $W_r + 0.5W_n$, where W_r is moisture at the plastic limit; W_n is natural moisture content). Taking into account that the layer of frozen soil at the time of the

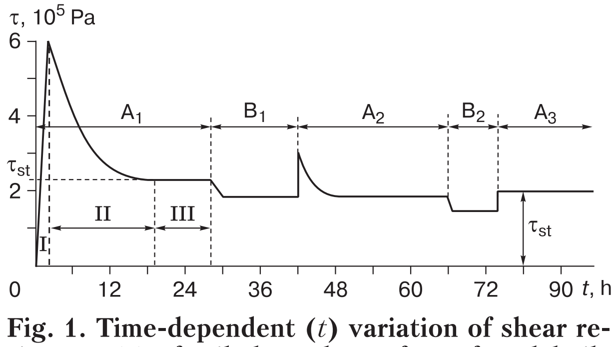

Fig. 1. Time-dependent (t) variation of shear resistance (τ) of soil along the surface of model pile frozen into soil.

τ_{st} – long-term shear resistance; A_1, A_2, A_3 – displacement at a rate of 10 mm/day; B_1, B_2 – no displacement; I, II, III – see the text for explanation.

initial shear has a temperature close to zero, and low thickness, its heave can not cause unallowable deformation of the foundation.

The *maximum adfreeze resistance as a design characteristic has thus poved of no practical value*. Rather the maximum value in the third stage (stage III) of the long-term adfreeze resistance should be taken as a design value (Fig. 1), given the site-specific permafrost-ground conditions [*Dubnov, 1967*].

The experiments of Yu.D. Dubnov showed that a consistent increase/decrease in the soil displacement rate on the foundation model does not change the value of the long-term adfreeze resistance, provided that soil displacement rate at each subsequent stage has shown not more that 10-fold variation. The contribution from the 50–100-fold variation of displacement rate is remarkable, inasmuch as it changes adfreezing resistance by 40–50 %. Given that naturally induced variations in the soil displacement along foundations are never abrupt, the influence of displacement rate while measuring long-term adfreeze resistance in the laboratory conditions can be ignored. To reduce the experiment time, the recommended rate is 10–20 mm/day [*Dubnov, 1967*]. According to the experiments of B.I. Dalmatov on ice-rich clayey soils, the long-term shear resistance was achieved at a shear rate of 10 mm/day and the total deformation of 1 cm.

Proceeding from this, it was proposed that the average value of tangential frost heaving forces is taken as equal to the magnitude of the long-term adfreeze strength. The stabilized shear resistance with respect to the frozen soil displacement along the model pile surface depends primarily on the temperature, and tends to increase as it decreases. The relationship is assumed to be linear until the temperature reaches −15 °C [*Dubnov, 1967*].

The freezing soil is heterogeneous throughout the depth, since its temperature varies from minimum on soil surface to the temperature of the inception of soil moisture freezing at the freezing front. When estimating the buckling load on foundations, it is necessary to take into account the temperature-dependent actual distribution of tangential frost heaving forces through depth. As such, this distribution was studied both in the on-site and laboratory conditions [*Dalmatov, 1957; Peretrukhin, 1967; Orlov, 1982*]. The most significant results of these studies are discussed below. The specific tangential frost heaving forces begin to increase from the onset of temperature decline in the frozen soil layer, and they tend to decrease when the temperature increases. In this case, the *specific* tangential frost heaving forces are maximum in the upper freezing layers (thickness: 0.2–0.3 m), while they show a decreasing trend with increasing freezing depth due to the temperature factor. The specific tangential frost heaving forces correlate with ultimately long-term shear resistance. However, the *integral* tangential frost heaving forces, depending on the total area of the soil-foundation contact, reach their maximum at a distance equal to 2/3 of the soil freezing depth (usually 1.0–1.5 m).

The data from foreign literature are consistent with the results of Russian researchers. Thus, Fig. 2 shows results of the field observations of temporal evolution of the vertical displacement of piles, tangential stresses and total frost heaving forces [*Penner, 1974*]. The freezing depth of clayey soil was 1.12 m,

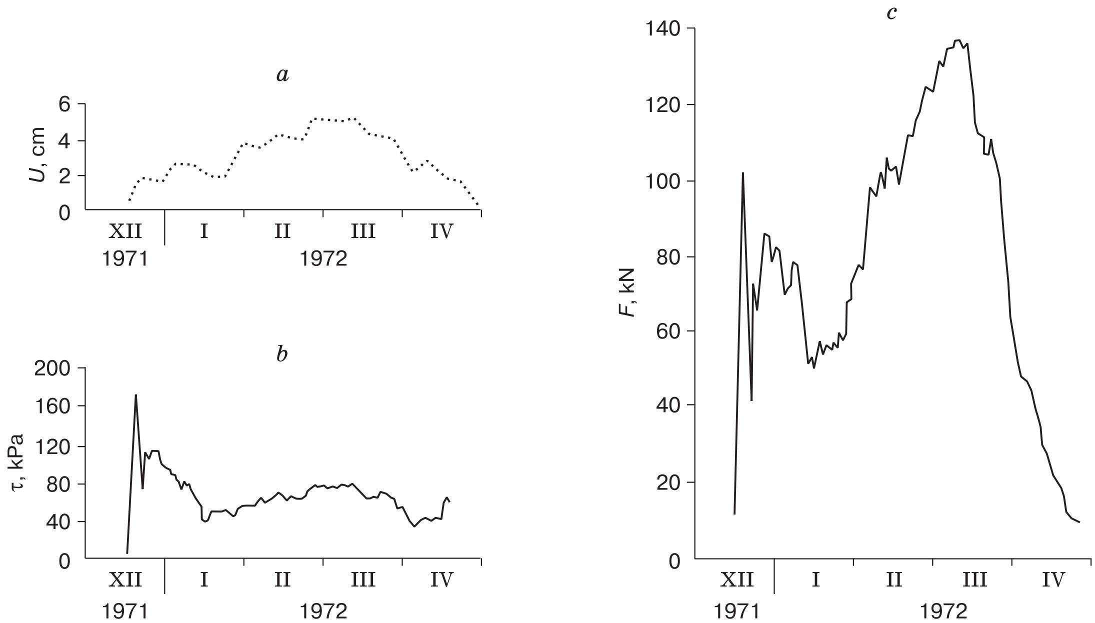

Fig. 2. Results of monitoring the vertical displacement with time of pile U (a), tangential frost heaving forces τ (b) and total (integral) frost heaving forces F (c) [*Penner, 1974*].

the metal pile was 32.4 cm in diameter. Results of the field observations revealed that the vertical displacement of pile *U* increases as a function (close to parabolic), from the values equal to zero in December to their maximum in the period from the end of February through mid-March, while the pile is gradually settled as the warming proceeded, regaining however its original position in late April–early May (Fig. 2, *a*).

The specific tangential frost heaving forces τ at the initial period of freezing (mid-December–early January) dramatically increase to 170 kPa, reaching the maximum and causing thereby a sheart-off along the adfreezing surface (Fig. 2, *b*). Then, until mid-January, they reduce to 40 kPa, followed by a slow growth up to 70 kPa and a change in the curve parallel to the parabolic time dependence of the pile displacement to 40 kPa by mid-April.

The total heave force *F* has a first peak (up to 100 kN) at the beginning of the freezing (mid-December), followed by a decline (to 50 kN) by mid-January, with subsequent long-term increase (to almost 140 kN) until mid-March (Fig. 2, *c*). Thus, the main provisions of the concepts of formation of tangential frost heaving forces represented by both Russian and foreign experts prove to be consistent with each other.

The construct of the method for laboratory determination of specific tangential frost heaving force of soils, presented in [*GOST R 56726-2015*], encompasses the physical aspects inferred from the published materials, as discussed above.

RESEARCH METHODS

The normative documents for engineering design of the Construction Code "Pile foundations" [*SP 24.13330-2011*] and "Bases and foundations on permafrost soils" [*SP 25.13330.2012*] provide calculus methods for determination of tangential frost heaving force from the magnitude of the specific tangential stresses, whose values are defined using the tabular data depending on the type and physical characteristics of soils.

The disadvantage of this method is that it neglects significant factors affecting the magnitude of the tangential frost heaving forces, including the soil temperature field dynamics during freezing and heaving. The 2015 GOST "Soils. Laboratory method for determining specific tangential forces of frost heaving" was developed with an aim of improving the technical regulations base in regard to determining tangential frost heaving forces [*GOST R 56726-2015; Cheverev and Alekseev, 2016*].

The GOST technical regulations recommend to derive specific tangential frost heaving forces in soils from the value of the long-term (stabilized) shear strength of the frozen soil specimen relative to the model foundation surface. To obtain the required data, the tests were carried out using a shear-strength apparatus ensuring the frozen soil failure (shear-off) along the adfreezing plane with a constant displacement rate 1–2 cm/day.

The experiments are carried out at three temperature values: –1, –2 and –6 °C. The long-term shear strength in the experiment is reported at the moment when the maximum displacement of the specimen of foundation material relative to the soil sample reaches at least 1 cm.

According to the Construction Code [*SP 25.13330.2012. Amendment No. 1*] tangential frost heaving forces acting along the lateral surface of the foundation are determined (relying on laboratory tests results) as is prescribed in [*GOST R 56726-2015*], in the manner described below.

1. The depth-dependent soil temperature variations are plotted for the end of the winter period until the phase change boundary (frost penetration depth). Soil temperatures are either measured at the construction site in accordance with [*GOST 25358-2012*] or determined by calculations, including numerical analysis methods (Fig. 3).

2. The temperature curve is divided into the three intercepts straddling: (1) from the freezing depth of the soil to the depth of established temperature at –1 °C; (2) from the depth at which the temperature is found to be equal –1 °C and to the depth where the temperature is –2 °C; (3) from the depth with the established temperature –2 °C to the ground surface temperature (GST).

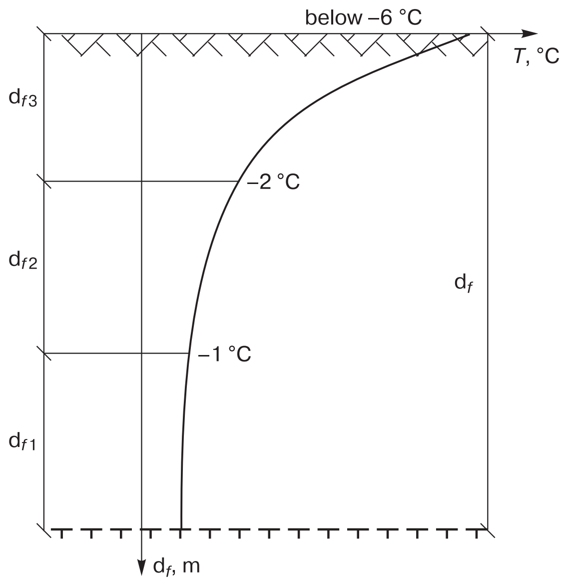

Fig. 3. Plot of soil temperature distribution through depth for calculation of tangential frost heaving force.

Thickness of frozen soil layer (d_f): d_{f1} – from the freezing point to –1 °C; d_{f2} – from –1 to –2 °C; d_{f3} – from –2 °C to ground surface.

3. The tangential frost heaving force will be equal to the sum of the products of the specific tangential frost heaving force obtained from laboratory tests for different temperatures (–1, –2, –6 °C) of soil adfreezing with lateral surface of the foundation, where soil temperature corresponds to the above values:

$$F_f = \tau_{fh1}A_1 + \tau_{fh2}A_2 + \tau_{fh3}A_3,$$

where τ_{fh1}, τ_{fh2}, τ_{fh3} are specific tangential frost heaving force (kPa) for temperatures: –1, –2 and –6 °C, respectively; A_1, A_2, A_3 are areas of the lateral surface of the foundation in the related zones.

To assess reliability of the methods for determining tangential frost heaving forces obtained in keeping with the technical regulations [*SP 25.13330.2012; GOST R 56726-2015*], the authors conducted a numerical simulation of temperature distributions through depth in winter season. The resulting temperature curves allowed determining the peaks of tangential frost heaving force, and plotting temporal and spatial variations of tangential frost heaving forces in winter season.

The temperature distribution through the depth of a soil layer was determined at the test site of the Research Institute of Bases and Underground Structures (NIIOSP) of the Research Center of Construction (Moscow) (Fig. 4). The measurements of frost heaving forces was carried out for a reinforced concrete pile (300 × 300 mm in the cross section) embedded in a hard-plastic clay loam; with the air temperature distributions for the 2011/12 winter period taken into account.

Tangential frost heaving forces were determined using the three methods discussed below.

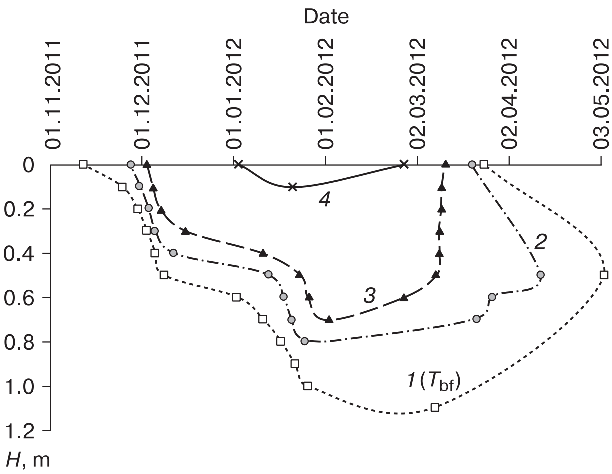

Fig. 4. Time-dependent variation of soil depth (*H*) for different temperatures: –0.2 °C (*1*), –1 °C (*2*), –2 °C (*3*) and –6 °C (*4*).

T_{bf} – temperature of the inception of soil freezing (freezing point).

Method 1. Soil freezing (frost penetration) depth was inferred from the temporal and spatial soil temperature distribution plots. Later, the tangential frost heaving force F_{fn} was calculated according to [*SP 25.13330.2012*] using the formula

$$F_{fn} = \tau_{fh}A_{fh},$$

where τ_{fh} is calculated specific tangential frost heaving force (kPa), taken in accordance with ii. 7.4.3 of technical regulations [*SP 25.13330.2012*]; A_{fh} is the area of soil adfreezing with lateral surface of the foundation within the modeled depth of seasonal freezing, m^2.

Method 2. The magnitude of tangential frost heaving forces was calculated in keeping with [*SP 25.13330.2012. Amendment No. 1; Cheverev and Alekseev, 2016*]. The values of specific tangential stresses or the strength of soil adfreezing with the foundation for temperatures –1, –2 and –6 °C were determined in accordance with the Construction Code [*SP 25.13330.2012. Amendment No. 1*] as a function of shear resistance along the soil-foundation adfreezing surface, which was plotted for different temperatures (Fig. 5).

Method 3. The tangential frost heaving forces were calculated according to the actual temperature distribution along the pile length with the distribution taken at each 0.5 °C. The values of adfreezing strength of the ground and the foundation surface were determined in compliance with the norms specified in [*SP 25.13330.2012*] (Figs. 4, 5):

$$F_{fn} = \sum_{i=1}^{df} \tau_{fh}A_{fh}.$$

The distribution of soil temperature in depth and time over the winter period was obtained using the FROST 3D Universal software. The plots of temperature distribution through depth (–0.2, –1, –2 and –6 °C) are shown in Fig. 4.

The values of time-dependent forces and shear stresses caused by frost heaving (adfreeze strength between soil and pile) were determined from the temperature distributions (Fig. 6). The calculations have

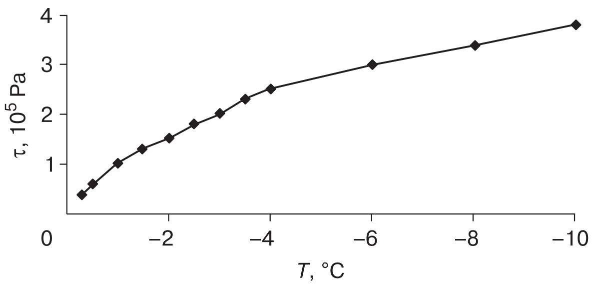

Fig. 5. Temperature-dependent shear resistance of frozen clay mortar along the foundation (according to the tabular data from [*SP 25.13330. 2012*]).

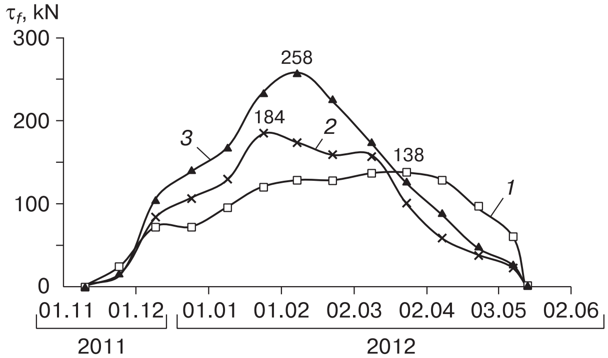

Fig. 6. Tangential frost heaving forces (τ_f) calculated by three methods, versus duration of freezing.

1 – Calculus Method 1; *2* – Calculus Method 2; *3* – Calculus Method 3.

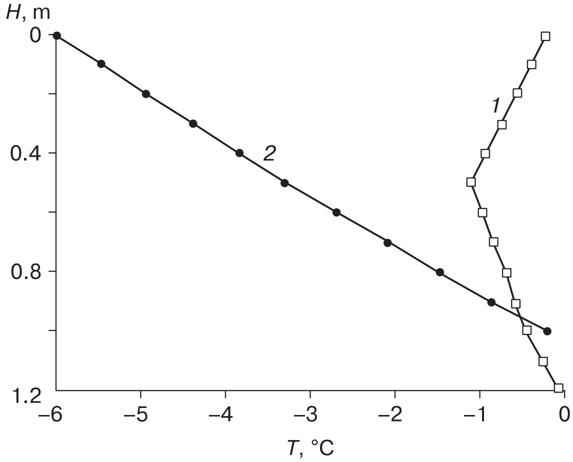

Fig. 7. Soil temperatures distribution in extreme periods through the depth of foundation embedment.

1 – end of winter season, 29.03.2012; *2* – the lowest temperatures distribution for winter season, 07.02.2012.

shown that when using Method 1 for the problem solving, the value of the tangential frost heave force increases with the deepening frost penetration, and reaches 138 kN.

Then, the values of time-dependent tangential frost heaving forces (soil and pile adfreeze strength) in were determined from the temperature distributions (Fig. 6). The calculations have shown that solving the problem with Method 1 the value of the tangential frost heaving force increases with frost penetration depth, reaching 138 kN.

The tangential frost heaving forces calculated by Method 2 constituted 52.8 kN for the end of winter season, when both the mean diurnal air temperature and the ground surface temperature tend to increase, despite the still proceeding soil freezing at depth.

At the maximum frost penetration depth the temperatures on the pile surface are minimal. Based on the analysis of temperature distribution through the depth of pile embedment, the temperature distribution graphs were plotted for the end of the winter season (late March) and for the winter season (early February) along the pile imbedded depth (Fig. 7). The value of the tangential frost heaving forces at the beginning of February was 184.8 kN, which is a 3.5-fold increase of the τ_f value at the end of winter season (Fig. 6). Figure 6 shows that as frost penetration depth increased with the affiliated lowering of soil temperature, the τ_f value explicably increases and begins to decrease as the temperature increases.

Method 3 yielded maximum averages of the tangential frost heaving forces, with the τ_f values peaking at 258 kN, which is explained by a significant decrease in soil temperature along the pile length. Whereas Method 2 revealed that shear resistance was interpreted as long-term only at temperature −2 °C in the temperature range between −2 and −6 °C within the given depth interval, suggesting that when Method 3 is applied these values will yield such values in greater amount, and they will be significantly higher at lower temperatures (−2.5, −3.0 °C, etc.).

CONCLUSIONS

1. The value of the tangential frost heaving forces (or long-term adfreeze strength) increases as the temperature decreases at the soil-foundation contact.

2. The tangential frost heaving forces derived from the specific tangential frost heaving force calculated according to the tabular data provided in the Construction Code [*SP 25.13330.2012*], yields significantly lower results (by about 26 %) compared to the laboratory GOST R 56726-2015 method [*SP 25.13330.2012. Amendment No. 1*].

3. The most accurate method for determining the magnitude of tangential frost heaving force is the laboratory method recommended by GOST R 56726-2015, applicable in all types of soils, with elementary layers of soil breakdown through the foundation's depth at temperature steps multiple of 0.5 °C. The values obtained by this method for determination of tangential frost heaving force were in 1.9 and 1.4-fold excess of those obtained according to ii. 7.4.3 of the Construction Code [*SP 25.13330.2012*] and [*SP 25.13330.2012. Amendment No. 1*], respectively. However, this method has proven to be time-consuming, since of the long-term strength of soil adfreezing with the foundation, at temperature steps multiple of 0.5 °C, requires a significant number of tests.

4. Finally, when determining tangential frost heaving forces, it is recommended to use the approach prescribed in the Construction Code [*SP 25.13330.2012. Amendment No. 1*] for determining the long-term adfreeze strength according to [*GOST R 56726-2015*] multiplied by the correction factor, which should be found either empirically from preliminary laboratory tests results or from plotting the function of temperature-dependent long-term adfreeze strength. The total value of the maximum tangential frost heaving force is determined from the worst distribution of temperatures through depth.

References

Bykov, N.I, Kapterev, P.N., 1940. Permafrost and Construction on it. Transzheldorizdat, Moscow, 372 pp. (in Russian)

Cheverev, V.G., Alekseev, A.G., 2016. Laboratory method for determining specific tangential forces of frost heaving (to draft GOST). In: Proceedings of the fifth conference of Russian geocryologists. MSU, Moscow, pp. 138–142.

Dalmatov, B.I., 1957. Effect of Frost heaving on Foundations of Structures. Gosstroiizdat, Leningrad; Moscow, 60 pp. (in Russian)

Dubnov, Yu.D., 1967. Laboratory tests of tangential forces of frost heave. In: Peretrukhin, N.A. (Ed.). Frost heave of soil and ways to protect structures from its effects. Transport Press, Moscow, pp. 55–60. (in Russian)

Guidelines for determining the physical, thermal and mechanical characteristics of frozen soils. 1973. Stroiizdat, Moscow, 191 pp. (in Russian)

GOST (State Standard) 25358-2012, 2013. Soils. Field method of determining temperature. Standartinform, Moscow, 16 pp. (in Russian)

GOST (State Standard) R 56726-2015, 2016. Soils. Laboratory method for determining specific tangential forces of frost heaving. Standartinform, Moscow, 6 pp. (in Russian)

Orlov, V.O., 1982. Evolution and field methods for calculation of tangential forces of frost heaving of soils. In: Rheology of soils and permafrost engineering. Nauka, Moscow, pp. 144–148. (in Russian)

Orlov, V.O., Dubnov, Yu.D., Merenkov, N.D., 1977. Heaving of Frozen Soils and Its Effect on the Foundations of Structures. Stroiizdat, Leningrad, 183 pp. (in Russian)

Penner, E., 1974. Uplift forces on foundations in frost heaving soils. Can. Geotechn. J., No. 11, 323–338.

Peretrukhin, N.A., 1967. The strength of frosty buckling of foundations. In: Frost heave of soil and ways to protect buildings from its effects. Transport Press, Moscow, pp. 25–54. (in Russian)

Roman, L.T., 2002. Mechanics of Frozen Soils. MAIK Nauka/Interperiodika, Moscow, 426 pp. (in Russian)

SP 24.13330-2011, 2011. Pile foundations. Published by FAI "Federal Center of Pricing in Construction and Building Materials Industry", Moscow, 85 pp. (in Russian)

SP 25.13330.2012, 2012. Bases and foundations on permafrost soils. Clause 7.4 (Amendment No. 1). Published by FAI "Federal Center of Pricing in Construction and Building Materials Industry", Moscow, pp. 28–30. (in Russian)

Tsytovich, N.A., 1928. On the question of the design of foundations erected on permafrost. Trudy Gipromeza, Leningrad, issue 2, pp. 3–14. (in Russian)

Tsytovich, N.A., 1973. Mechanics of Frozen Soils. Vysshaya Shkola, Moscow, 446 pp. (in Russian)

Vyalov, S.S., Egorov, N.I., 1958. Experimental Evaluation of Frost Heave Forces. Trudy Instituta Merzlotovedenia, USSR Academy of Sciences, vol. 14, 40–56. (in Russian)

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