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THAW DEPTHS IN PERMAFROST SOILS UNDER ROAD EMBANKMENTS IN THE PRESENCE OF HEAT INSULATION ON SLOPES

A.A. Zhang

Lomonosov Moscow State University, Faculty of Geology, Department of Geocryology, 1, Leninskie Gory, Moscow, 119991, Russia; zhang.andrew.msu@gmail.com

Thaw depths in permafrost soils under road embankments are calculated analytically with a new method based on approximation of numerical solutions to the 2D Stefan problem. The method allows choosing the required thickness of heat insulation on the embankment slopes to provide the stabilization of the underlying permafrost.

Embankment, permafrost, stabilization, heat insulation, calculation, data approximation, Brandon method

INTRODUCTION

A method for preventing years-long thawing of permafrost under railway and motor roads was suggested in 2008 [*Ashpiz et al., 2008a,b*]. It consists in laying a geosynthetic heat insulator on embankment slopes, which increases seasonal frost depth above the active layer and causes a cooling effect [*Tsytovich, 1928*]. However, the insulation is poorly efficient at small snow depths. The geosynthetic insulator (extruded polysterene foam, XPS) is laid under the slope base and is fixed on the slope with metal pins. The method was tested successfully at a segment of the Amur-Yakutsk railway in 2009 through 2016 [*Zhang et al., 2018*].

Till recently, thermal resistance to heat insulation has been calculated numerically using the available software for solving 2D Stefan's problem, which is time-consuming in some cases. This paper presents an analytical solution based on approximation of numerical data.

NUMERICAL SOLUTION OF A 2D STEFAN PROBLEM

The 2D Stefan problem was solved numerically using the WARM software designed at the Department of Geocryology in the Moscow State University [*Emelyanov et al., 1994*], using data from three weather stations in different permafrost areas (Chum, Salekhard, and Amga) as boundary conditions. The simulation provided estimates of thaw depths beneath the road shoulder after the stabilization of mean annual ground temperatures (steady state). The input data used for calculation of boundary conditions and thermal parameters of soils are summarized in Table 1.

The thermal resistance of heat insulation on the slopes should be such that the thaw depth of the embankment did not exceed its height at the shoulder, which is a critical point next to the slope as a heat source (Fig. 1). In the steady state case, the thaw depth at this point depends on nine parameters: the

Table 1. **Input data for calculating boundary conditions on the surface of modeling domain and thermal properties of soil**

Weather station	Ω_s	Ω_w	$R_{s,or}$	$R_{w,or}$	$R_{s,de}$	$R_{de,w}$	R_{ins}	T_0	λ_{th}	λ_f	L_v	T_{bf}
Chum	41 172	–76 504	0.102	0.243	0.102	4.14	0–3.5	–0.5	1.97	2.32	20 925	–0.1
Salekhard	38 617	–84 607	0.050	0.227	0.050	4.06	0–2.5	–1.0	1.97	2.32	20 925	–0.1
Amga	73 146	–130 305	0.103	0.249	0.103	2.14	0–2.0	–2.5	1.97	2.32	20 925	–0.1

Note: Ω_s is the sum of summer air temperatures corrected for solar radiation, °C·h; Ω_w is the sum of winter air temperatures, °C·h; $R_{s,or}$, $R_{w,or}$, $R_{s,de}$, $R_{w,de}$ are, respectively, the summer and winter values of thermal resistance to air-road surface heat exchange within slopes, m^2·°C/W; R_{ins} is the thermal resistance to heat insulation on slopes, m^2·°C/W; T_0 is the calculated soil surface temperature in natural conditions, °C. The thermal properties of roadbed soil: λ_{th}, λ_f are the thermal conductivity coefficients in unfrozen (thawed) and frozen roadbed, W/(m·°C); L_v is the volumetric heat of soil moisture phase transitions, W·h/m^3; T_{bf} is the freezing-thawing points of soils, °C.

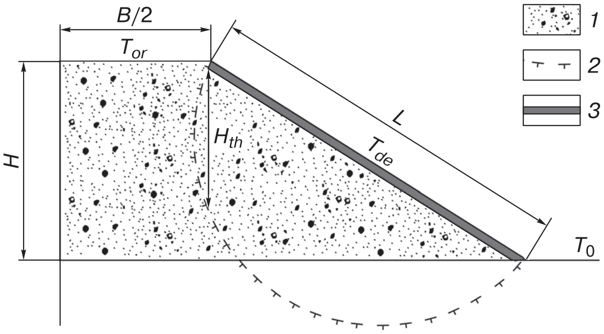

Fig. 1. Sketch of a roadbed.

1 – gravel-sand soil; *2* – thaw boundary beneath slopes; *3* – heat insulation on slopes. Symbols are explained in text.

system size (B, H, L), boundary conditions (T_{bf}, T_{or}, T_{de}, T_0), and thermal conductivity (λ_{th}, λ_f):

$$H_{th} = (T_{bf}, T_{or}, T_{de}, T_0, \lambda_{th}, \lambda_f, B, H, L), \quad (1)$$

where λ_{th} and λ_f are thermal conductivity coefficients of unfrozen (thaw) and frozen embankment soils, respectively, in W/(m·°C); T_{bf} is the freezing point (temperature at which soil freezing begins), °C; T_{or} is the calculated temperature of the embankment surface (carriage way plus shoulders), °C; T_{de} is the calculated temperature of the slope surface, °C; T_0 is the calculated soil surface temperature in natural conditions, °C; B is the crest width, m; H is the embankment height, m; L is the slope length, m.

The calculated soil surface temperature T_0 is a mean annual soil temperature at the active layer base, assuming that the soil is infinite.

In order to reduce the number of variables in (1), it can be presented in a dimensionless form using the difference $T_{bf} - T_{or}$ as a temperature scale (temperatures are counted from T_{bf}) the height H as a scale of linear dimensions. Then the relative thaw depth beneath the shoulder ξ_{sh} is

$$\xi_{sh} = \frac{H_{th}}{H} = f\left(\frac{2\lambda_{th}\left(T_{de} - T_{bf}\right)L}{\lambda_f\left(T_{bf} - T_{or}\right)B}, \frac{T_{bf} - T_0}{T_{bf} - T_{or}}, \frac{B}{H}, \frac{L}{H}\right). \quad (2)$$

The ratio L/H can be excluded from variables at a slope of 1:n (n = 1.5), which is the most frequent case in road engineering. Then, the relationship becomes a function of three variables:

$$\xi_{sh} = f(\alpha, \beta, \gamma), \quad (3)$$

$$\alpha = \frac{2\lambda_{th}\left(T_{de} - T_{bf}\right)L}{\lambda_f\left(T_{bf} - T_{or}\right)B}, \quad \beta = \frac{T_{bf} - T_0}{T_{bf} - T_{or}}, \quad \gamma = \frac{B}{H}.$$

The boundary conditions T_{or} and T_{de} were calculated using previously reported empirical equations [*Zhang et al., 2018*] for blowing snow and frost areas distinguished according to the mean winter wind speed v_w [*Construction Norms and Regulations, 2012*].

Table 2. **Numerical solutions to 2D Stefan problem for long-term thaw depths beneath road shoulder**

α	β	γ	ξ_{sh}^{m}	ξ_{sh}^{cl}	δ
5.15	0.40	2	4.72	4.15	12.08
2.98	0.48	2	2.91	3.12	7.31
4.50	0.53	2	3.75	3.68	1.71
6.84	0.53	2	4.82	4.45	7.68
2.12	0.58	2	2.33	2.57	10.36
1.95	0.40	2	2.86	2.68	6.23
1.42	0.88	2	1.0	1.97	96.93
1.82	0.88	2	1.77	2.20	24.51
2.70	0.88	2	2.79	2.63	5.87
1.66	0.22	1.5	2.63	2.40	8.88
2.18	0.22	1.5	3.2	2.71	15.21
3.22	0.22	1.5	4.0	3.23	19.34
5.30	0.22	1.5	5.05	4.04	19.97
4.06	0.47	1.5	4.16	3.05	26.72
4.55	0.47	1.5	4.43	3.21	27.53
1.74	0.83	1.5	1.50	1.85	23.37
2.02	0.83	1.5	1.88	1.98	5.56
2.78	0.22	1	2.23	2.40	7.32
3.64	0.22	1	2.62	2.71	3.326
5.38	0.22	1	3.19	3.23	1.08
8.85	0.22	1	3.96	4.04	2.10
6.74	0.47	1	3.38	3.04	10.13
7.56	0.47	1	3.57	3.20	10.26
1.87	0.83	1	0.99	1.52	53.27
2.77	0.83	1	1.75	1.81	3.31
3.21	0.83	1	2.13	1.93	9.35
4.55	0.83	1	2.65	2.26	14.69
3.70	0.22	0.75	1.95	2.31	18.69
4.86	0.22	0.75	2.27	2.62	15.23
7.17	0.22	0.75	2.74	3.12	13.70
11.80	0.22	0.75	3.38	3.90	15.45
8.98	0.47	0.75	2.97	2.94	1.06
10.08	0.47	0.75	3.12	3.09	0.94
2.50	0.83	0.75	1.35	1.47	8.53
3.69	0.83	0.75	1.95	1.75	10.47
4.28	0.83	0.75	2.11	1.87	11.40
6.07	0.83	0.75	2.63	2.18	16.79

Note: α, β, γ are dimensionless parameters; ξ_{sh}^{m}, ξ_{sh}^{cl} are the relative thaw depths beneath road shoulder found numerically and analytically; δ is the approximation error, %; mean error is 14.77 %.

For areas of blowing snow (v_w > 4.6 m/s):

$$T_{or,\,de} = -5.513M_{or,\,de} + 6.64, \quad (4)$$

for frost areas ($v_w \leqslant 4.6$ m/s):

$$T_{or,\,de} = -7.762M_{or,\,de} + 7.68, \quad (5)$$

where v_w is the mean winter wind speed, m/s; 5.513, 6.64, 7.762, 7.68 are constants, °C; $M = d_f/d_{th}$ (the dimensionless variable M is a ratio of the winter frost depth to the summer thaw depth as a parameter of the ground thermal state suggested by *Tsytovich* [*1928*]); d_f, d_{th} are, respectively, the largest possible depths of seasonal frost and thaw of roadbed, m.

To use the correlation relationship, one has to know local seasonal frost and thaw depths in the construction area. They can be found in the first approximation using Stefan's equations:

$$d_{th}=\sqrt{\frac{2\lambda_{th}\Omega_s}{L_v}+\left(\lambda_{th}R_s\right)^2}-\lambda_{th}R_s; \qquad (6)$$

$$d_f=\sqrt{\frac{2\lambda_f\left(-\Omega_w\right)}{L_v}+\left(\lambda_f R_w\right)^2}-\lambda_f R_w; \qquad (7)$$

$$R_s=\frac{1}{\alpha_s}+R_{ins}; \qquad (8)$$

$$R_w=\frac{1}{\alpha_w}+R_{ins}+R_{snow}, \qquad (9)$$

where L_v is the volumetric heat of phase transitions in soil moisture, W·h/m^3; Ω_w is the sum of winter air temperatures (absolute value), °C·h; Ω_s is the sum of summer road surface temperatures, °C·h, calculated according to [*Construction Norms and Regulations, 2012*]; R_s, R_w are, respectively, the mean summer and mean winter thermal resistances to heat transfer on the soil surface within the crest and slopes, m^2·°C/W; α_s, α_w are, respectively, the summer and winter heat transfer coefficients of the crest and slope surfaces (convective heat transfer on the surface), W/(m^2·°C); R_{ins} is the thermal resistance to heat insulation laid on slopes (assuming R_{ins} = 0 on the traveled way surface), m^2·°C/W; R_{snow} is the mean winter thermal resistance of snow on the crest and slopes, m^2·°C/W. The details of the method were described by *Khrustalev* [*2005*].

The numerical solutions are presented in Table 2 as α, β, and γ dependences of the dimensionless parameter ξ_{sh}.

APPROXIMATION OF NUMERICAL SOLUTIONS

The thaw depths obtained with equation (2) are approximated by an elementary analytical function using the method of Brandon, an American statistician [*Brandon, 1959; Polyakov, 2008*]. The method ensures satisfactory approximation accuracy and is useful to describe processes quickly and precisely, based on experimental data. The procedure consists of several steps:

1. Denoting the calculated relative thaw depths beneath the road shoulder ξ_{sh}^m as y and the dimensionless coefficients α, β, and γ as x_1, x_2, and x_3, respectively: $y = f(x_1, x_2, x_3)$.

2. Obtaining normalized y values for each of 37 variants: $y_0=y/\overline{y}$, where $\overline{y}$=const is the mean arithmetic y value (2.83 m).

3. Finding the $y_0 = f(x_1)$ relationship from three functions: $y = a + bx$ (linear), $y = a + b \ln x$ (logarithmic), and $y = ax^b$ (exponential), and using the most accurate relationship in further calculations. The minimum approximation error δ is given by

$$\delta=\frac{\left|y-y_0\right|}{y_0}\cdot 100\ \%.$$

In our case, the error is the smallest (26.42 %) with the exponential function $f_1\left(x_1\right)=0.577x_1^{0.45}$.

4. Estimating the conventional criterion y_1 for each x_2 as $y_1=\frac{y_0}{f_1\left(x_1\right)}$. This residual criterion depends on x_2 values rather than on x_1:

$$y_1=\frac{y}{\overline{y}f_1\left(x_1\right)}=f_2\left(x_2\right).$$

After solving the equation $y_1 = f_2(x_2)$, the relationship with the smallest δ is chosen. This is again the exponential function with an average error of 25.15 %: $f_2\left(x_2\right)=0.745x_2^{-0.213}$.

5. Obtaining the new criterion y_2 which depends on x_3

$$y_2=\frac{y_1}{f_2\left(x_2\right)}=\frac{y}{\overline{y}f_1\left(x_1\right)f_2\left(x_2\right)}=f_3\left(x_3\right),$$

calculating $y_2 = f_3(x_3)$ as above, and again choosing the relationship with the smallest δ, which is the exponential function with an average error of 14.77 %: $f_3\left(x_3\right)=0.903x_3^{0.571}$.

6. Calculating the relative thaw depth beneath the shoulder as $y=\overline{y}f_1\left(x_1\right)f_2\left(x_2\right)f_3\left(x_3\right)$.

Finally, the dependence of this thaw depth on the dimensionless variables α, β and γ becomes

$$\xi_{sh}^{cl}=1.1\alpha^{0.45}\beta^{-0.213}\gamma^{0.571}. \qquad (10)$$

The thaw depth beneath the shoulder is found as

$$H_{th}=H\xi_{sh}^{cl}.$$

The approximation errors δ for modeling results according to equation (10) are listed in Table 2.

EXAMPLE CALCULATION

Problem formulation. A railroad embankment in the area of Labytnangi Community (weather station Salekhard, blowing snow area) has a crest width of B = 6 m, a height of H = 4 m, and a slope of n = 1.5. The heat exchange between the embankment surface and air is α_s = 20 W/(m^2·°C) in summer and α_w = 16.7 W/(m^2·°C) in winter. Mean winter thermal resistances of snow on the shoulder and slopes are, respectively, $R_{snow,\,or}$ = 0.167 m^2·°C/W and $R_{snow,\,de}$ = = 4 m^2·°C/W. The sum of embankment surface temperatures, corrected for solar radiation, is Ω_s = = 38 617 °C·h in summer and Ω_w = –84 607 °C·h in winter. The embankment material has a thermal conductivity of λ_{th} = 1.97 W/(m·°C) when unfrozen and λ_{th} = 2.32 W/(m·°C) when frozen. The volumetric heat of soil moisture phase transitions is L_v = = 20 925 W·h/m^3; the freezing point is T_{bf} = –0.1 °C. The initial soil temperature at the active layer base is T_0 = –1 °C.

The calculations aim at estimating the thermal resistance of heat insulation laid on the slopes which would provide the maximum thaw depth beneath the road shoulder H_{th} no greater than the embankment height H.

Calculation procedure. The calculations are performed by iteration until the condition $H_{th} \leqslant H$ has been satisfied. At the first step, the thermal resistance of the slope to heat insulation is assumed to be absent: R_{ins} = 0. Then the slope length is found as

$$L=\sqrt{H^2+(nH)^2}=\sqrt{4^2+(1.5\cdot 4)^2}=7.21 \text{ m}$$

and the thermal resistance values on the ground surface in the summer and winter time are then calculated with (8) and (9), respectively, for the embankment top and slopes: $R_{s,or} = R_{s,de} = \frac{1}{20}$ = 0.05 m^2·°C/W, $R_{w,\,de} = \frac{1}{16.7} + 4$ = 4.06 m^2·°C/W, $R_{w,\,or}$ = $= \frac{1}{16.7} + 0.167$ = 0.22 m^2·°C/W.

At the next step, the seasonal thaw depths beneath the embankment top and slopes are found with (6):

$$d_{th,or}=d_{th,de}=\sqrt{\frac{2\cdot 1.97\cdot 38\,617}{20\,925}+(1.97\cdot 0.05)^2}\; - \; 1.97\cdot 0.05 = 2.6 \text{ m}$$

and then the seasonal frost depths beneath the embankment top and slopes are estimated with (7):

$$d_{f,or}=\sqrt{\frac{-2\cdot 2.32\cdot(-84\,607)}{20\,925}+(2.32\cdot 0.23)^2}\; - \; 2.32\cdot 0.23 = 3.84 \text{ m};$$

$$d_{f,de}=\sqrt{\frac{-2\cdot 2.32\cdot(-84\,607)}{20\,925}+(2.32\cdot 4.06)^2}\; - \; 2.32\cdot 4.96 = 0.95 \text{ m}.$$

Correspondingly, the ratios of the two values, the dimensionless parameter M on the embankment top and slopes are $M_{or}=\frac{3.84}{2.6}=1.48,\ M_{de}=\frac{0.95}{2.6}=0.36.$

Further calculations are performed with equations (4), (3), and (10):

(4) for the embankment surface temperature within the top and slopes: T_{or} = –5.51·1.48 + 6.64 = = –1.49 °C; T_{de} = –5.51·0.36 + 6.64 = 4.63 °C;

(3) for the dimensionless parameters

$$\alpha=\frac{2\cdot 1.97\cdot(4.63-0.1)\cdot 7.21}{2.32\cdot(-0.1+1.49)\cdot 6}=6.937,$$

$$\beta=\frac{-0.1+1}{-0.1+1.49}=0.647,\ \ \gamma=\frac{6}{4}=1.5;$$

(10) for the dimensionless thaw depth beneath the shoulder (10): ξ_{sh} = 1.1·6.937$^{0.45}$·0.647$^{-0.213}$× ×1.5$^{0.571}$ = 3.61.

Thus estimated thaw depth turns out to be much greater than the embankment height (H_{th} = = 3.61·4 m >> 4 m), i.e., the required condition fails. Thus, heat insulation is laid on the slope and the iteration repeats at different R_{ins} until the condition $H_{th} \leqslant H$ fulfills.

Note in conclusion that the condition $H_{th} \leqslant H$ fulfills at the thermal resistance R_{ins} = 4.55 m^2·°C/W which gives H_{th} = 3.7 m, or 0.3 m smaller than the embankment height.

CONCLUSIONS

The method of analytical calculations for the depth to permafrost (thaw depth) under the road embankment is based on Brandon approximation of numerical solutions to the 2D Stefan problem. According to this method, the dependence of thaw depth on thermal resistance to heat insulation laid on the slopes can be presented in a dimensionless form for three variables: dimensionless temperature of top (crest) and slopes of the embankment and its width-to-height ratio.

The obtained relationship can be used to choose the required thickness of insulation on the slopes which would ensure thermal stabilization of roadbeds, which is illustrated by the example calculations.

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