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ICE COMPLEX STRUCTURE INFERRED FROM VERTICAL ELECTRICAL SOUNDING (SOUTHERN MARGIN OF THE LENA-AMGA INTERFLUVE)

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Discussed are the results of a cluster analysis of vertical electrical sounding (VES) data with an aim to characterize the Ice Complex structure in its southern margin stretching along the Berkakit–Tommot–Yakutsk railway corridor (the Tommot–Kerdyom section). In this area, the geoelectrical section is strongly variable and generally reflects a 5-layer structure of the Ice Complex. Ice contents are higher in the upper 30 m of the Ice Complex (horizons 1–3) composed of very ice-rich silts with relict syngenetic ice wedges. Horizons 4 and 5 occurring lower (to depths of 50–60 m) contain massive ice bodies of unknown origin, which does not rule out the presence of firn ice in Quaternary–Tertiary sediments. The data obtained by vertical electrical sounding and electromagnetic profiling indicate a strong decrease in volumetric ice content in the upper 10–15 m silts (705-th track kilometer) at the boundary between the Maganskaya and Emilskaya terraces in the Lena-Amga watershed, which is the southern end of the vast alas province (thermokarst basins) of the low to medium-height terraces along the right bank of the Lena River valley. A drastic reduction in the amount of relict massive ice bodies in the upper layers of the southern margin section and their complete disappearance in lower part of the section occur at the 697-th km point, on the top of the Emilskaya terrace. Within a 3 km distance further south, the presence of ice bodies in the upper 10 m from the terrace surface is ranked as sporadic, with the occurrence probability of 0.1 %.

Boreholes, Ice Complex, massive ice, vertical electrical sounding, geoelectric section, cluster analysis, electrical resistivity

INTRODUCTION

The Ice Complex (IC) of the Lena-Amga interfluve, a major geomorphological element of the Central Yakutia lowland, is distinguished by cryogenic singularity and complexity of the geological structure. In the mid 20th century, large-scale research projects bringing together science and industry were implemented within the interfluve area, with the Zheldorproekt Yakut expeditions and those commissioned by the Central Administrative Authority of Construction of Railway Lines subordinated to the Ministry of Internal Affairs (MVD) of the USSR (1948–1951), as well as by the Academy of Sciences of the USSR (1950–1954), and Geological Prospecting Agency (1951–1954) being the most representative with respect to territorial scope and range of activities. The participation of some well-known permafrost researchers (A.A. Grigoriev, N.A. Grave, A.I. Efimov, P.A. Soloviev, P.A. Shumsky, N.I. Ivanov, I.N. Votyakov) and geophysicists (L.A. Dobrovolsky, B.N. Dostovalov, R.I. Korkina and others) in the expeditions appeared to have formative influences on the research program.

With all the extensive factual material on the structure of the Lena-Amga interfluve accumulated during the expeditions, the diversity of the researchers' opinions on the nature of the Ice Complex did not challenge the understanding of the presence of massive ice as its main distinction. The collective viewpoint was formulated by P.A. Soloviev [*1959*], interpreting the Ice Complex as "*a discrete, more or less uniform horizon abounding in ice veins and having sheet-like geometry over the vast areas, whereas its age, composition, origin and thickness are varying*" (*p. 49*).

While analyzing and systematizing the collected (sometimes disparate) factual materials obtained during the expeditions, P.A. Soloviev arrived at the conclusion about the predominant distribution of wedge ice bodies (ice wedges) in the Ice Complex of the Lena-Amga interfluve. Moreover, based on the geomorphological and landscape affinity between two large – Lena-Amga and Lena-Vilyuy – interfluves of the Central Yakut Lowland he assumed a possibility of the ubiquitous distribution of wedge ice bodies over the entire alas province comprising the low- and medium-height terraces in the midstream Lena river valley [*Soloviev, 1959, p. 4*].

As such, this assumption is well justified for the upper part of the IC which was well studied to a depth of 5–7 and 10–15 m during the aforementioned fieldworks, whereas the nature of relict massive ice in the middle and lower parts of the IC still remains unknown and may not be associated with the widely distributed wedge ice bodies. Thus, in 2005 and 2011, during well drilling (by the staff of Melnikov Permafrost Institute (MPI) SB RAS) on the elevated portion of the northern margin of the Lena-Amga interfluve, the boreholes which penetrated the upper layer of wedge ice bodies subsequently exposed the lower (base) layer of massive ground ice of firn origin (i.e. formed by firn densification) [*Spektor et al., 2011*]. Locally, the Ice Complex may have a similar or more complex multi-layered polygenetic structure within the interfluve area. Whether this is or is not the case we will see from future scientific studies, however for the time being, the question of the IC structure and thickness either on individual layers (horizons) or throughout the section of Quaternary sediments remains unanswered, despite its being highly topical and particularly relevant in respect to the increased pace of industrial and agricultural development of the Central Yakut Lowland.

This paper set out to investigate the IC structure and thickness both at discrete horizons and across the section of Quaternary sediments. For this purpose, a statistical analysis of the data obtained at the southern edge of the Lena-Amga Ice Complex by vertical electrical sounding (VES) was performed.

This traditional land-based geoelectric technique is used not only for detailed characterization of the boundaries of inhomogeneities in the geoelectric section (for which the electrical resistivity tomography (ERT) method has been advantageously used), but also for solving engineering-geological and other related problems of the structural analysis of the section changing with depth, laterally and over time. The importance of this method applications to tackling the problem of permafrost engineering (e.g. predicting the state of frozen soils) was emphasized by V.P. Melnikov [*1977*]: “We need to be adequately informed as to how the parameters of the electromagnetic field characterize the entire section of a site, area, landscape type, etc., rather than individual elements of the target interval alone. At this, VES curves types can serve as the evaluation criteria” (*p. 60*).

The advantage of such an integrated approach utilized as its extended version for studying the subsurface structure was highlighted by B.S. Svetov and M.N. Berdichevskii, drawing a certain analogy between the stationary electric and gravimagnetic methods as part of geophysical prospecting. Bearing in mind the unity of integral nature of the electric, gravitational and magnetic fields, these outstanding geophysicists welcomed an opportunity to refute redundancy in obtaining data, rather focusing on a comprehensive study of their independent parameters [*Svetov and Berdichevskii, 1998*].

STUDY AREA

On the small-scale permafrost landscape map of Yakutia giving area characterization with respect to construction on permafrost, the Central Yakut Lowland encompasses as many as three interfluves formed by the Lena river: the Lena-Vilyuy (in the northeast), and the Lena-Aldan and the Lena-Amga (in the north) [*Melnikov, 1968*]. In the vast area of the latter, the bedrock base of the Lena river valley is composed of Paleozoic and Mesozoic rocks, overlain ubiquitously by predominantly alluvial Cenozoic sediments. The thickness of alluvium is found variable within the Quaternary and Tertiary intervals, reaching 60–100 and 200–300 m, respectively [*Soloviev, 1959, p. 19*]. The permafrost thickness in the interfluve along the Lena Highway varies, according to the electric well logging and VES data, from 400 m (Bestyakhskaya terrace of the Lena river) to 230–260 m (the Kachikattsy village area) [*Kalinin and Yakupov, 1989, p. 90*].

The Ice Complex (IC) within the right-bank Maganskaya terrace of the Lena river valley (part of the site investigated by VES technique) is underlain by sedimentary rocks comprisoing Jurassic sandstones, siltstones, as well as Cambrian limestones, dolomites, occurring at a depth from 80 to 120 m. The overlying variously grained sands with pebbles are topped by sand-loams, clay-loams and clays with pebbles [*Soloviev, 1959, p. 24*]. Results of the geotechnical investigations revealed Quaternary deposits (depth: 10–20 m and below) composed mainly of silty-oozy and clay- and sand-loamy lithologies. These formations represent the parent environment for ice wedges, the probability of encountering which, for example, on the middle-altitude right-bank Abalakhskaya terrace of the Lena river valley equals 30–88 % [*Soloviev, 1959, p. 43*]. Besides, ice wedges are observed on a higher Maganskaya terrace of the Lena valley.

Ice wedges and their host sediments formed during the Karganian time (marine isotopic stage, MIS-3) and Sartanian (MIS-2) time of the Late Neopleistocene [*Ivanov, 1984; Grinenko et al., 1995*]. The geophysical survey site is located on the 694–707-th km stretch between Tommot and Kerdyom railway stations (Fig. 1), where the axis of the projected Berkakit–Tommot–Yakutsk railway line crossed the southern edge of the forested IC of Lena-Amga interfluve as of the period of VES surveys. The geotechnical investigations of this problematic construction site were commissioned by Projecttransstroy OAO with an aim to clarify the IC structure, and to provide insights as to whether the Ice Complex has completely or partially pinched out (a topical issue to both designers and geocryologists), with the youngest right-bank moderate-altitude Maganskaya terrace passing into the higher, erosion-accentuated Emilskaya terrace[1] of the Lena valley. In other words, this being the Lena and Amga rivers interfluve which is inferred from the landscape and geomorphological zoning to be a transition from the alas-valley region to the polygenetic plain subregion [*Varlamov et al., 2006*]. Note that geocryologists suggest delineating the southern boundary of the Ice Complex specifically in this place, proceeding from the speculative thermokarst origin of alases[2] [*Abolin, 1929*].

The IC structure was fairly exhaustively studied by the VES technique at the construction site (the 694–707-th km stretch) during the 2005 Yakut Prospecting&Survey Expedition (YAPSE), which also included drilling of 104 core drill holes (depth: to 5–20 m; spacing: 150–200 m) along the Amur-Yakut Mainline (AYAM) axis, with soil sampling for the laboratory analysis.

The drilling data revealed that the subsurface section is composed of clay-loams with rare sand interlayers. According to the YAPSE results, deposits temperature in the lower part of the zero annual amplitude layer (depth: 10 m) varied from –1.3 to –4.8 °C, averaging –3.2 °C. Bedrocks were exposed only by six wells on the Emilskaya terrace, which are described as follows: bedrock (limestone) in the 9.2–9.8 m depth interval (BH No. 506, 507, 524, within the 693.7–694.1 km stretch); compacted sand with limestone clasts at depths of 3.8 and 9.6 m (BH No. 563, 696.2 km point); white sand and sandstone at 9.1 and 11.8 m (BH No. 564, 696.3 km point), while yellow orange sand and strongly-ferruginized red-brown sandstone at depths of 3.6 and 17.3 m (BH No. 565, 696.1 km point).

The YAPSE drilling results have shown that a south-trending (toward Tommot village) decrease in thickness of the Ice Complex entailed changes in its composition, the origin of dispersive soils, as lacustrine-alluvial loams of the Maganskaya terrace become replaced with eluvial-deluvial loams and sands of the Emillskaya terrace. In this area, the IC consists of two horizons, composed of high porosity, dispersed soils (mainly clay-loams), occurring within the 1–3 and 6–8 m depth intervals. At the 95 % confidence level, a random spread of the boundary depth was ±(0.1–0.4) and ±0.5 m.

According to the laboratory analysis data, ice content of soils of clayey composition at the expense of visible ice inclusions (i) and volumetric ice content in the first and second horizons changed in the range of 27.4–77.7 and 25.0–87.7 %, respectively, with their arithmetic mean values (45.8 ± 2.2) and (52.9 ± 4.3) %; while soils are classified as very ice-rich [*State Standard, 2018*]. For AYAM, the coefficients of variation (CV) of index i and occurrence depth of the boundary of the 1st and 2nd horizons of very ice-rich and extremely ice-rich (i > 60 %) dispersed soils are 20–27 %, while variability of the horizons' thickness (on average 2 m) reaches 71 %.

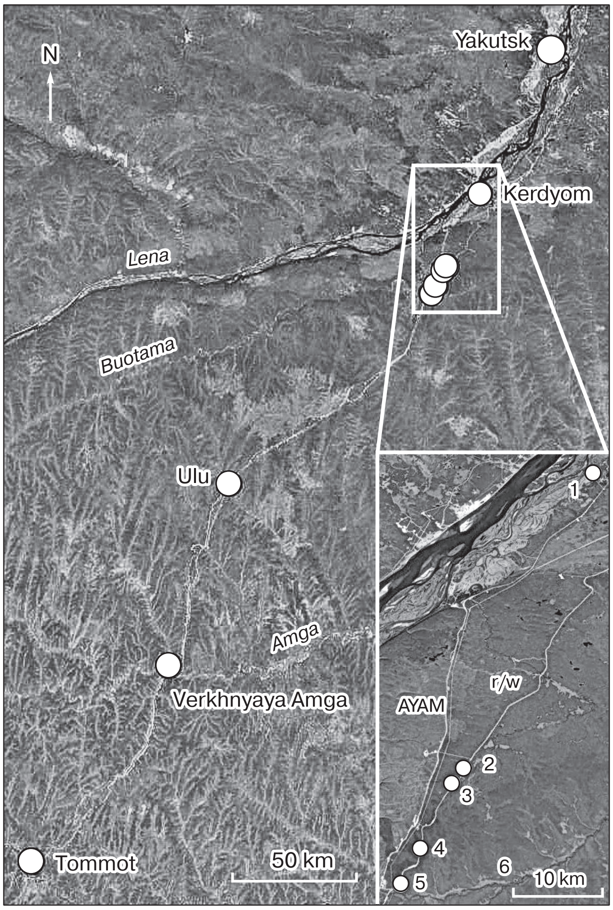

Fig. 1. General layout of the study area on the satellite image and its VES survey-covered stretch along the railway line (the right inset), in the vicinity of the Lena Highway:

1 – Kerdyom village; 2 – 707-th km of the railway line (end of the study area, VES point 51); 3 – 705-th km of railway line (Maganskaya and Emilskaya terraces junction); 4 – 697-th km (southern nearly entirely pinch-out boundary of the IC); 5 – 694-th km (beginning of the study area – VES point 1); 6 – Verknyaya Lyutenga river.

Of all the boreholes drilled within the scope of the YAPSE activities, fragments of ice wedges (interlayers and layers of ice with vertical veins of the host soil) were found at 44 well points. This means that the fraction of ice wedges in the section of the pro-

[1] This and other, higher standing terraces are cited as denudated and, in themselves, appear to be small-sized outliers localized on the watersheds of transitional polygenetic plakor plains.

[2] This notion is viewed by local Yakut people as closed and semi-closed cover collapse sinkholes (with or without a lake in the bottom) which develop in the middle of the forest on large expanses of flat interfluves [*Bosikov, 1991, p. 6*].

Fig. 2. Fragment of an extensive outcrop of ice wedge polygons at the AYAM reconstruction site reported in 2011 near the village of Kachikattsy.

Photograph by L.G. Neradovskii.

jected railway line studied by the VES method is 40 % (as inferred from the drilling data).

From the statistical analysis of the YAPSE drilling data it follows that at the 95 % confidence coefficient, wedge ice bodies, very ice-rich and extremely ice-rich soils form three-layer structure of the IC with their depths, accordingly, averaging 2.5–5.8, 4.6–7.9, 8.5–10.1 m and a spread of single values in the range ±(0.1–0.8), ±(0.6–1.2), ±(3.0–2.9) m. The 1^{st} uppermost horizon is more common than lower horizons (the occurrence probability for of the 1^{st} horizon is 25 %, while for the 2^{nd} and 3^{rd} – 18.3 and 4.8 %).

The CV values for the upper/lower boundaries occurrence depth and wedge ice thickness in horizons 1–3 amount to 14/34/61, 28/36/85, 28/22/83 %, respectively. These values allow to deduce that wedge ice (as very ice-rich and extremely ice-rich soils) in the three horizons are less dynamic with respect to their occurrence depth, against thickness. As such, this paradox can be seemingly explained by statistical nuances, in particular, by the fact that the variations of single values indicating wedge ice thickness are comparable with its mean values.

Thus, according to the YAPSE drilling operations, the southern margin of the IC of Lena-Amga interfluve has a complicated structure viewed as a combination of two horizons of very ice-rich and extremely ice-rich soils with three-layer structure of wedge ice, which tends to be encountered more often in the first (uppermost) horizon of very ice-rich and extremely ice-rich soils, than in the lower horizons. Moreover, in 78 % of the cases, the YAPSE boreholes accidentally exposed transverse apophyses or “shoulders” of ice wedges varying from 0.1–1.0 and 1.4–4.6 m in height. Only in 7 (25 %) of the cases, boreholes tapped the middle part of wedge ice body (7–9 m thick) or adjacent lateral part 4–5 m in thickness.

Along the projected railway line (the 693–736-th km stretch), the distribution of wedge ice aggradation localities was previously studied (before VES surveys) by the dipole electromagnetic profiling (DEMP) method to a depth of 10 m in the first horizon of the IC of the Lena-Amga interfluve [*Neradovskii, 2008*].

The intricate configuration of wedge ice bodies at a depth of about 3 m overlain by clay-loams, is illustrated in Fig. 2 as a fragment of a rare outcrop of the first horizon (with the presence of wedge ice) of the Ice Complex on the Maganskaya terrace of the Lena-Amga interfluve during the Lena highway reconstruction in the area of its proximity to the operating Tommot–Kerdyom–Nizhny Bestyakh railway corridor, at a distance of about 1–3 km from the northern limit of the study area.

RESEARCH METHODS

The VES surveys methodology is best described in the instructions for electrical prospecting [*Frantov, 1984, p. 129–136*] given in textbooks authored by A.I. Zaborovskiy, Yu.V. Yakubovsky, V.K. Khmelevsky, etc. It will therefore be suffice to do with a brief description, for general understanding of the specifics of the field works and VES data processing method employed in the on-site study of the Ice Complex in the Lena-Amga interfluve.

The VES surveys were carried out in the summer of 2006 using Schlumberger array with maximum half way spread length (AB/2) from 0.3 to 220 m. The distance between VES points was 200 m. Electric current and voltage in source and receiver lines with a length of 0.2 and 6.0 m were measured by the AE-72 electronic autocompensator. The voltage to current measurements transition[3] from the first to the second receiver line was made at AB/2 = 9 (depth: 15 m).

A set of household round batteries complete with special anode batteries (100 AMTsG-U-190CH) served as an autonomous portable DC power supply, capable of changing output voltage stepwise within a wide range (from 1.5 to 400 V) in the transmitter circuit.

Results of the field measurements (VES curves) were processed using the IPI2W software as instructed by the User Manual [*2001*]. The authors’ methodology applied at the start of the data processing enabled automatic recognition of the model type of geoelectric section through setting the optimum minimal number of layers. This was followed by interactive selection to determine the model type of geoelectric section with automatic calculation of its parameters which included: electric resistivity (ER), occurrence

[3] Such a simple scheme of electric sounding with a maximum half way spread distance 1000–1500 m was developed in the late 1960s and 1970s at the Permafrost Institute of the USSR Academy of Sciences in order to study the thickness of frozen strata to a depth of several hundred meters.

depth of the top and bottom boundaries of electric layers and their respective thicknesses.

The data on geological and cryogenic structure of the IC in the Lena-Amga interfluve available from previous studies simplified the searches for reliable VES curves interpretations in order to infer the type and parameters of layers in the geoelectric section. The knowledge of the seasonally thawing layer (active layer) thickness measured using a metal probe, along with the phenomena of the apparent resistivity (AR) maxima indicative of proximity to relatively high-resistivity layers in the homogeneous loamy stratum contributed to the calculation accuracy of the layer parameters. Given the invariable lithogenetic processes in the IC complex, high-resistivity layers with a high degree of probability corresponded to depth intervals with high weighted average values of volumetric ice content due to the increasing content of texture ice in dispersed soils, and numerous ice wedges.

The occurrence depth of the upper near-horizontal boundary of wedge ice bodies was determined by drilling and the VES surveys, with account of local or background anomalies determined by the DEMP method. However, for accurate determination of the lower boundary position of wedge ice thickness with intent to capture the maximum penetration depth of the ramified network of small ice veins (a few tens of a centimeter in thickness), neither drilling nor geophysical methods would work.

There are other constrains limiting the accuracy of determinations of structural parameters of individual horizons or the entire IC, specifically: (1) methodological, concerning the traditional in geotechnical investigations irregular patterns of selective soil sampling from drill cores[4] for laboratory analysis of the frozen soils properties, e.g. their ice content; (2) technical, which is associated with a lack of standardized and adequate metrological tools for well logging to a depth of 10–20 m.

Given the unavailability of more complete data on the ice content and electrical resistivities of frozen soils distribution with depth, approximate estimates of both aggregate thickness of the IC and its discrete horizons were derived from the averaged position of their unified lower boundary, delineating the relict massive ice masses in bulk, regardless of their origin. As such, the integrated approach was laid down by P.A. Soloviev in his definition of the Ice Complex [*Soloviev, 1959*] and N.A. Grave, who proposed the method of areal mapping of the IC boundaries as a single geocryological unit with the surface expression as its landscape and geomorphology, rather than diagnosing ice wedge thicknesses alone [*Grave, 1944*].

GEOELECTRIC SECTION

The problem of classification, or typification, of the geoelectric section structure was solved by analyzing configuration of the VES curves. Judging from this characteristic attribute, the geoelectric section of the studied site of the IC is composed of 3–5 layers with varied ER values distribution along the lateral and with depth. In rare cases (probability 1–8 %) the structure of geoelectric section was determined to be the types marked as: H, K, HA, HK, KH, HAK, AKQ, KQH, KHK, AKHK. Most often, its structure was classified as type A, AK, AKH, HKH. The deciphering and explanations of the abbreviations can be found in the classical textbooks on electrical prospecting authored by A.I. Zaborovskiy, Yu.V. Yakubovsky and L.L. Lyakhov, V.K. Khmelevsky et al. [*Zaborovskiy, 1963; Yakubovsky and Lyakhov, 1988; Khmelevsky and Shevnin, 1994*].

Among the factual material consisting of a total of 65 VES points, high resistivity layers of the geoelectric section which contain the element marked “K” were deduced from relative maximum apparent resistivity values on the VES curves and constitute the biggest portion (71 %). The geoelectric section is constructed with account of absolute elevations (Fig. 3, *c*), the section of AR values (Fig. 3, *b*) and total longitudinal conductivity graphs (Fig. 3, *a*). The latter are based on three depth intervals (pseudosections), selected in such a way that soil electrical conductivity dynamics provided insights about the distribution of the volume of relict (massive) ice in the IC with depth and along the AYAM mainline.

The first pseudosection of the geoelectric section is set to a depth of 10 m, i.e. the depth at which the IC is investigated mainly by small geotechnical wells. The IC studied in detail by drilling to this depth is viewed as wedge ice body made up by three horizons. A high degree of dispersive soils (mainly clay loams) saturation with massive ice (ice wedges) in the upper part of the complex translates to low conductivity of the geoelectric section in the first slice.

On the second pseudosection set to a depth up to 50 m, electrical conductivity of dispersive soils (dominantly sandy-clayey, lacustrine-alluvial by origin) increases, which is attributed to a decrease in the total volume of relict massive ice in the IC middle and lower portions.

The third pseudosection is set to a depth of 100 m proceeding from general understanding of the geological structure of the IC of Lena-Amga interfluve, i.e. to a depth interpreted as most likely for relict massive ice of polygenetic nature to be missing from the Quaternary–Tertiary boundary deposits passing into sedimentary rocks to compose the basement of Maganskaya and Emilskaya terraces. Soil

[4] Each lithology type is represented by one sample.

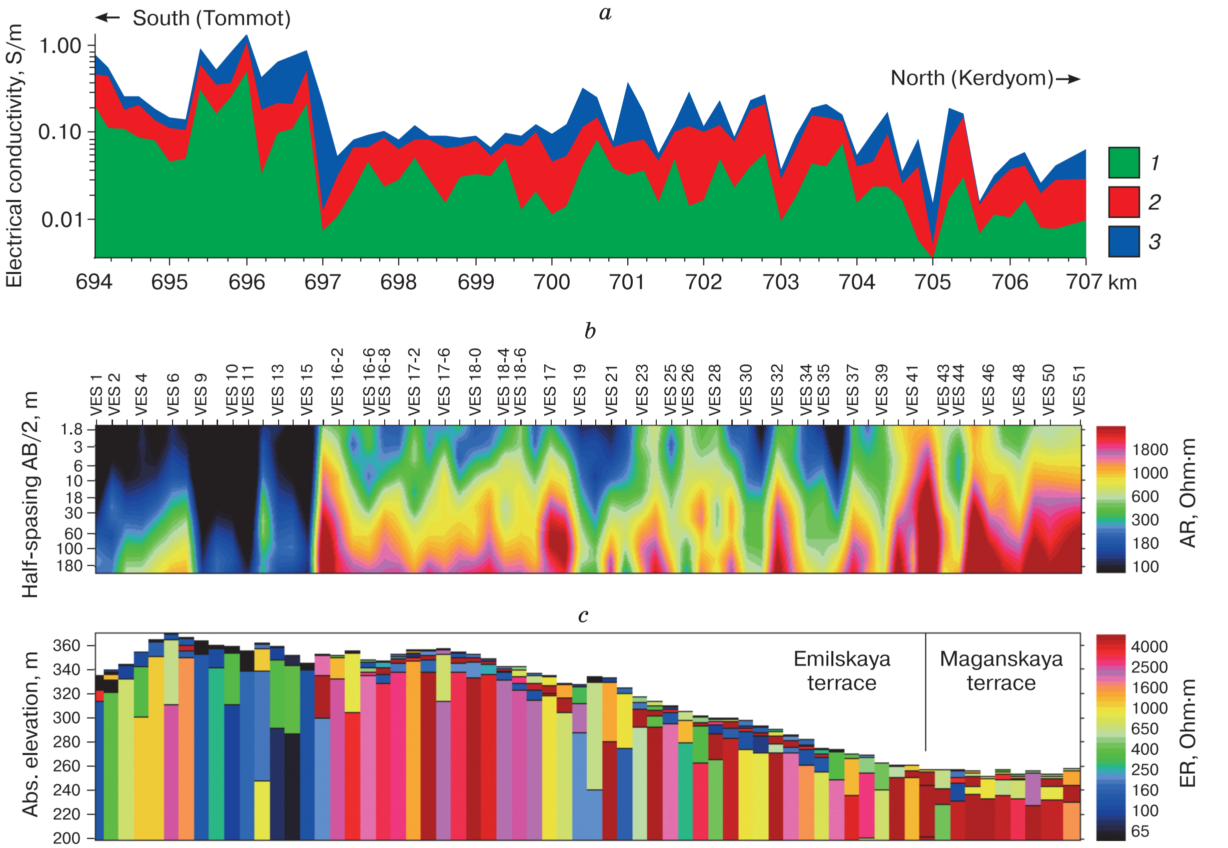

Fig. 3. Ice Complex of the Lena-Amga interfluve at the location of the projected railway line (694–707-th km):

a – total longitudinal electrical conductivity of soils within three depth intervals: *1* – 0–10 m, *2* – 0–50 m, *3* – 0–100 m; *b* – apparent electrical resistivity section (AR), AB/2 – half-way spread; *c* – geoelectric section. The VES curves are processed using the IPI2Win software.

conductivity continues to increase in this part of the section, however not as much as expected according to the assumption of completely degraded relict massive ice.

The discrepancy between the expected and observed electrical conductivity behavior is accounted for the bedrocks having high resistivity, which is comparable to resistivities of the undivided layer of ice soils and ice. According to the data obtained by geophysicist R.I. Korkina, who conducted the pioneering research in the Lena-Amga interfluve, ER values for limestones and sandstones composing the basement of the Tyungyulyunskaya and Abalakhskaya terraces locally reach 8,000–20,000 Ohm·m [*Korkina, 1949*].

In the studied portions of the Maganskaya and Emilskaya terraces, maximum ER values of the layers composing the lower part of the geoelectric section are close to those obtained by R.I. Korkina (6800–8500 Ohm·m). Their averages are lower, though, but comparable to resistivities of very ice rich and extremely ice-rich soils and the wedge ice bodies in uppermost horizons of the IC (2125–2765 Ohm·m).

There are three important observations deduced from the analysis of Fig. 3, namely: (1) a high differentiation of the geoelectric section structure along the AYAM mainline; (2) the background AR values tend to grow with depth; (3) soil conductivity abruptly increases southwardly (towards Tommot r/w station) first at the 705-th km (2-fold), then even more (almost 5-fold) at the 697-th km. This distinction is none the less expressly seen in the AR section (Fig. 3, *b*).

The first and second distinctions reflect the structural-petrophysical heterogeneity of the IC along the lateral and with depth, while the third feature (locally manifested at the 705-th km) is linked to the Maganskaya and Emilskaya terraces junction, where the disappearance of thermokarst lake basins (alas basins) occurred concomitantly with massive ice thawing out.

The available integrated geological and geophysical materials represented by landscape-geomorphological zoning data, YAPSE drilling data, and results of the VES and DEMP methods, revealed a sharp decrease in ice content in the uppermost Ice Complex

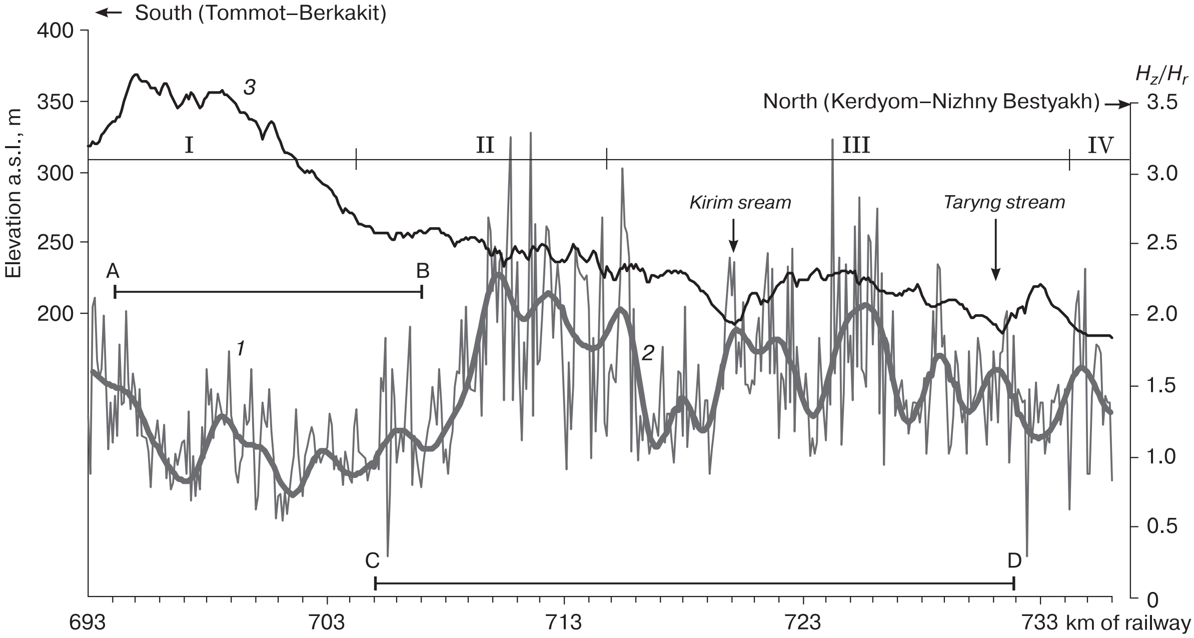

Fig. 4. Results of the studies of the Ice Complex of Lena-Amga interfluve at locations of Tommot–Kerdyom stations of the projected Tommot–Berkakit–Yakutsk railway line [*Neradovskii, 2008, p. 463*].

I–IV – terraces of the Lena river valley: I – Emilskaya; II – Maganskaya; III – Abalakhskaya; IV – Tyungyulyunskaya. *1* – initial values of H_z/H_r; *2* – smoothed filtered (background) values H_z/H_r; *3* – earth's surface relief; A–B – boundaries of the study area determined by VES method; C–D – Ice Complex boundaries delimiting high volumetric ice content (to a depth of 10 m) at the expense of large concentrations of wedge ice bodies distributed laterally.

at 705-th km. This is best observed from the DEMP data to a depth of 10 m on the graphs showing a relationship between the vertical (H_z) and horizontal (H_r) components of the harmonic field of the vertical magnetic dipole (frequency: 1.125 MHz) (Fig. 4). The results were obtained by the DEMP method with the receiving and transmitting antennas spaced at 20 m apart and with the distance 10 m between the measurement points along the axis.

Judging from the lowering unevenness (dispersion[5]) of the conductivity plot on the pseudosections of depths to 50 and 100 m, most of the IC lower portion localized in the 3–5-m horizons of wedge ice and other ice of polygenetic nature (below 10–20 m), extends to a distance of 700 km, with the ice content showing a decreasing trend. Here, topography of the Emilskaya terrace slope becomes slightly less elevated, with the incision depth reaching 10 m. The lower part of the IC pinches out and the content of relict massive ice dramatically decreases, varying from negligible to non-existent at the top of the terrace (697-th km) where an extensive trough as deep as 20 m presumably formed as a result of the denudation processes in one of the large tectonic depressions. According to the YAPSE drilling data and those obtained by Melnikov Permafrost Institute, occurrence of discrete ice wedges at the 697-th km point and further is not ruled out, though it can be described as a fragmentary disorderly pattern with a very low probability (about 0.1 %).

Major important characteristics of the geoelectric section of the southern margin of the IC within the Lena-Amga interfluve are reflected in the composite VES curve of the AKH type[6], obtained from actual material comprising measurements in 65 sounding points (Fig. 5). Given that the median version of the generalization translates to more accurate configuration of the composite VES curve for the predominant type of geoelectric section, Fig. 5 therefore shows the median values of the geoelectric section parameters. The integrated high-resistivity layers of the geoelectric section is interpreted as the upper (wedge ice thickness) and lower (sediments of ice of the unknown nature) ice-rich horizons of the IC. The upper horizon with thickness measuring 5.4 m at ER = 8500 Ohm·m occurs at a depth of 7.1–12.5 m. Taking into account the known equivalence principle, the position of its boundaries varies from 7.2–7.4 m

[5] Measure of the geoelectric section structure heterogeneity in horizontal range.

[6] Similar VES curves were observed in the north of the Lena-Amga interfluve, within the alluvial plain of Pleistocene age in the Lena-Tatta interfluve [*Melnikov and Niskovskikh, 1978, pp. 62–64*].

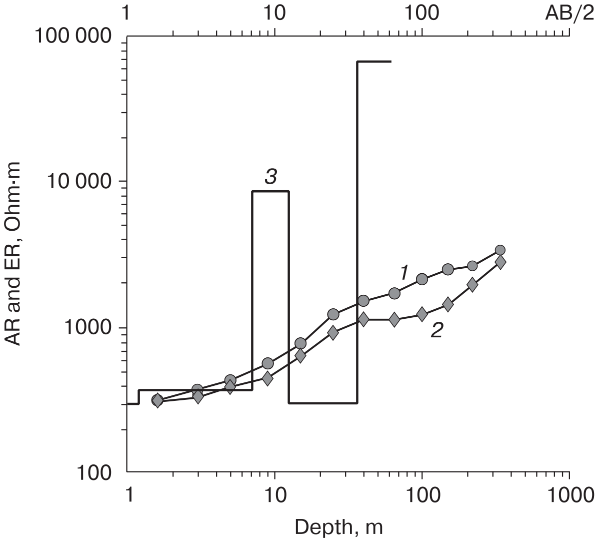

Fig. 5. Integrated VES curves averaged by arithmetic (*1*) and median (*2*) metric.

3 – boundaries occurrence depth and aggregate thicknesses of composite geoelectric section of the southern margin of the Ice Complex of Lena-Amga interfluve.

with anomalously high ER values (ER = = 390,000 Ohm·m) to 6.5–23.5 m with low ER values (ER = 2900 Ohm·m).

The second variant of the parameters equivalence in the upper horizon of the composite geoelectric section is more realistic, despite the low ER values yielded by the contained ice[7]. Contributed by the VES data obtained by R.I. Korkina on the Abalakhskaya terrace, the objectivity of this estimate increases. According to these, the upper part of the IC consists of four wedge ice bodies hosted by clay-loams within two depth intervals: 1.4–4.7 and 5–10 m [*Korkina, 1949, p. 16*]. The upper boundary of the lower horizon of the IC in the composite geoelectric section occurs at a depth of 36 m with ER = 65 800 Ohm·m. The lower boundary depth of the lower horizon remains unknown.

CLUSTER ANALYSIS OF THE GEOELECTRIC SECTION

In contrast to classification of a type of geoelectric section deduced from the VES curves, the application of cluster analysis allows to more objectively evaluate (from the standpoint of mathematical statistics) the IC southern margin structure within the Lena-Amga interfluve and more accurately determine the number of ice horizons composing it and their occurrence depth.

The numbers of VES points served as the objects of clustering, while the variables represented the geoelectric section parameters: the type, depth of the upper boundary (top) and lower boundary (base) of high resistivity layers, along with their thickness and resistivity. The ratio between the number of objects and the number of variables which equals 10 and more satisfies the condition of correct clustering of inputs. This problem was solved using the STADIA software [*Kulaichev, 2006, pp. 350–365*]. Prior to clustering, the original set of 65 VES points was divided into two regions: inclusive and exclusive of AR maxima in the upper, middle and lower parts of the VES curves.

The descriptive statistics of the parameters of high-resistivity layers of the geoelectric section was derived from the clustering results (Table 1).

Table 1. **Descriptive statistics of integrated parameters of clusters of high-resistivity layers 1–5 in the geolectric section**

Statistics of the VES points clusters	Cluster 1				Cluster 2				Cluster 3				Cluster 4		Cluster 5	
	M_1	M_2	H	ER	M_1	M_2	H	ER	M_1	M_2	H	ER	M_1	ER	M_1	ER
Arithmetic mean (*AP*)	10.3	30.0	19.5	17310	7.6	27.7	21.0	4221	4.1	7.5	3.4	4427	18.7	2527	60.3	3068
Standard error *AP*	1.4	4.5	3.9	4923	1.0	6.4	5.7	1448	0.5	0.7	0.3	640	1.4	276	12.0	975
Median value (*ME*)	9.9	26.5	14.0	7444	6.8	14.8	7.0	1952	4.6	8.1	3.1	3545	18.4	2272	53.5	2039
Mean modal value	–	–	14.0	–	–	–	–	–	3.3	11.5	–	–	18.5	2972	–	–
Standard deviation	4.8	15.7	13.4	17053	3.9	25.4	22.7	5792	2.2	3.1	1.2	2933	8.9	1743	34.0	2758
Minimum	4.6	10.1	5.5	357	2	0	0	364	0.8	1.9	1.1	499	4.3	134	25.5	816
Maximum	17.6	55.7	42.6	47086	14.8	68.6	56.7	21289	8.3	13.7	5.6	11539	50.6	6774	114.0	8505
Coefficient of variation, %	46.3	52.2	68.6	98.5	51.3	91.7	108.0	137.2	53.6	41.3	35.3	66.2	47.6	69.0	56.5	90.0
Number of VES points	12	12	12	12	16	16	16	16	21	21	21	21	40	40	8	8
Confidence level *AP* 95 %	3.0	10.0	8.5	10835	2.1	13.5	12.1	3086	1.0	1.4	0.6	1334	2.8	557	28.5	2305

Note. M_1, M_2 are occurrence depths of the upper and lower boundary of high-resistivity layers, m; H is thickness of high-resistivity layers, m; ER is electrical resistivity, Ohm·m.

[7] These can not be high, given that the VES method, when involving a large volume of sampling focuses on ice wedges polygons (as a system) partitioned by intervals of clayey soils with their inherent enhanced or high salinity, rather than on a discrete ice wedges alone. Besides, each ice wedge will contain (to any extent) organic residues and interlayers of host rocks.

Table 2. **Composite geolectric section and Ice Complex structure inferred from the clustered VES points**

Cluster No.	Horizon No.	Average parameters of integrated clusters (horizons)							
		M_1		M_2		H		ER	
		AP	*ME*	*AP*	*ME*	*AP*	*ME*	*AP*	*ME*
3	1	4.1	4.6	7.5	8.1	3.4	3.1	4427	3545
2	2	7.6	6.8	27.7	14.8	21.0	7.0	4221	1952
1	3	10.3	10.0	30.0	26.5	19.5	14.0	17300	7444
5	4	60.3	53.5	–	–	–	–	3068	2039
4	5	18.7	18.4	–	–	–	–	2527	2272

Note. M_1, M_2, H are occurrence depths of the upper/lower boundary and thickness of high-resistivity layers in the section (ice horizons), respectively, m; ER is electrical resistivity, Ohm·m; *AP*, *ME* are arithmetic mean and median value.

To determine the geoelectric section structure, its average parameters were ordered in the direction towards the increasing occurrence depth of the upper interface of high-resistivity layers. The result of this ranking operation is shown in Table 2, and plots illustrating changes in the mean values of the occurrence depth of the boundaries and ER clusters in the layers (horizons) are given in Fig. 6. It is shown that the IC southern margin of Lena-Amga interfluve consists of five horizons, whose average thickness increases with their occurrence depth. According to averaged estimates rounded to the whole number, the three uppermost wedge ice horizons occur within the IC at depths given below: from 4–5 to 7–8 m (ER= 3500–4400 Ohm·m) for the 1st horizon; from 7–8 to 15–20 m (ER = 1900–4200 Ohm·m) for the 2nd horizon; from 10 to 27–30 m (ER = 7400–17 300 Ohm·m) for the 3rd horizon.

The lowermost and deepest 4th and 5th horizons containing massive ice of unknown nature occur within the 18–19 and 54–60 m depth interval (ER = = 2000–3100 Ohm·m). Their lower boundary depth has been detected neither by drilling nor VES surveys. The nature of relict massive ice bodies occurring at great depths remains unclear either. According to the MPI research findings, the possibility of their syngenetic glacial-firn origin with occurrence of disparate horizontally stratified ice horizons[8] is postulated.

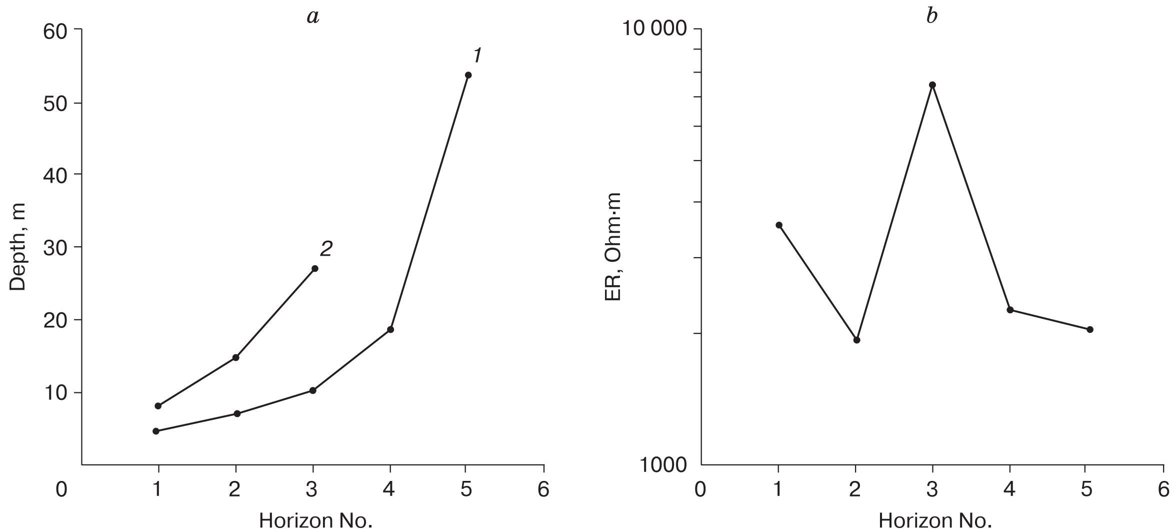

Fig. 6. Average values of the occurrence depth of boundaries of the Ice Complex of Lena-Amga interfluve (*a*) and ER of high resistivity layers of the geoelectric section (*b*) in the ranked numbers of horizons of the southern margin of the IC.

1 – upper boundary; *2* – lower boundary.

[8] In the north of the Lena-Amga interfluve with absolute elevations (220–250 m) similar to the study area, firn ice with various degree of crystallinity is found to be buried beneath wedge ice bodies (2.5–5.0 m) within the depth intervals indicated below: 12.0–17.0, 23.3–24.5, 33.6–39.0 m [*Spektor et al., 2011*].

DISCUSSIONS

Discussion of the VES investigations results obtained in the southern margin of the IC of Lena-Amga interfluve has prompted two major questions as to (1) where the southern border of the Ice Complex is and (2) what kind of structure it has.

The first question was answered by joint consideration of independent factual material obtained by different researchers. On the one hand, these were the materials of landscape and geomorphological zoning of the Lena-Amga area of interfluve, and on the other hand, results obtained by the VES and DEMP methods.

Analysis of these integrated geological and geophysical materials allowed an inference that the pinch-out boundary of the bulk massive ground ice as aggraded bodies of wedge ice and ice of other nature in the upper and lower parts of the Ice Complex is confined to the top of the Emilskaya terrace of the Lena valley (Fig. 7, *a*)[9].

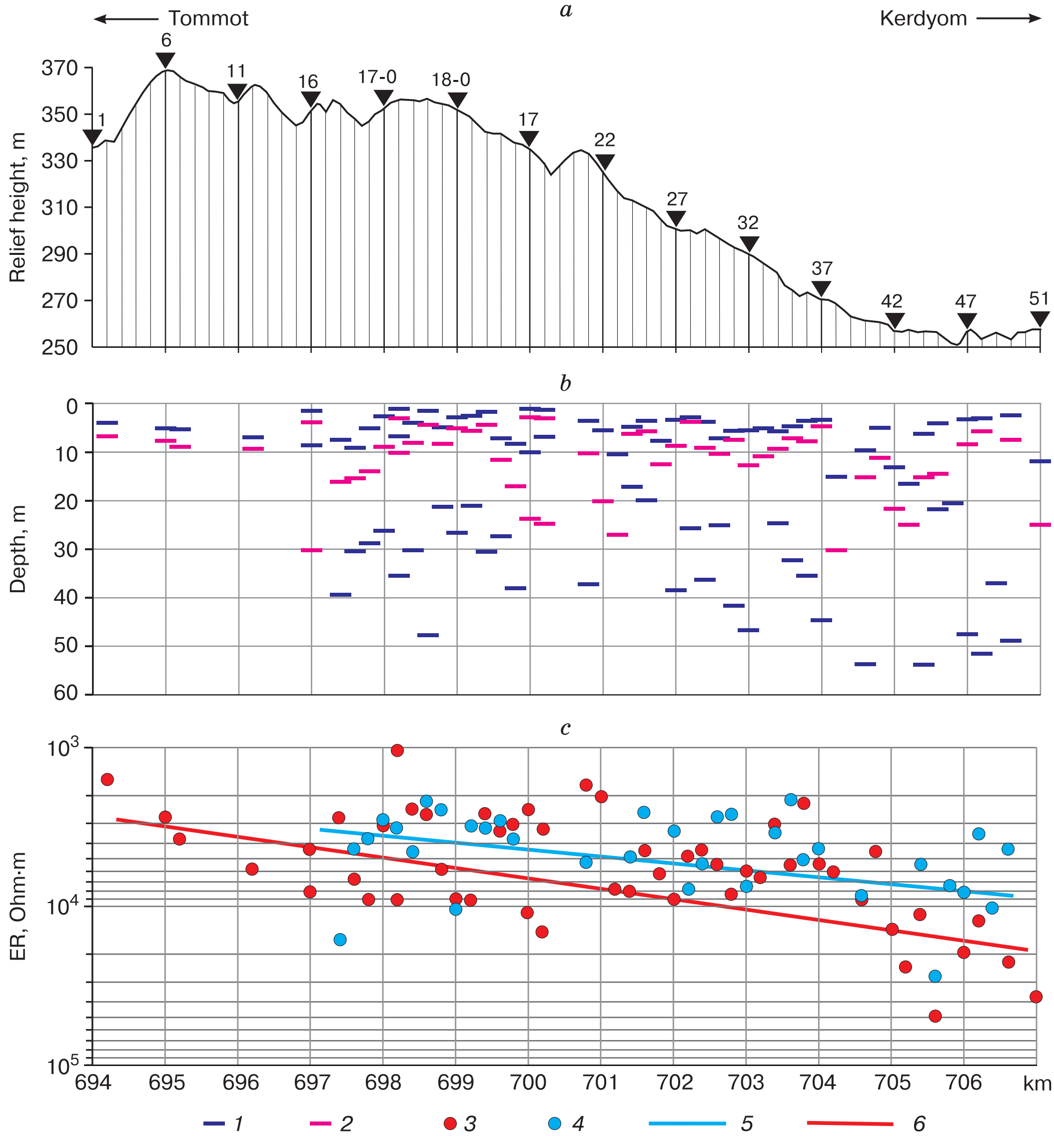

Fig. 7. Ground ice within the southern margin of the IC distribution area in the Lena-Amga interfluve.

a – site topography studied by the VES method (1–51 – numbers of VES points); *b* – point distribution fields of occurrence depths of the upper (*1*) and lower (*2*) boundaries of high resistivity layers of the geoelectric section; *c* – ER of high resistivity layers corresponding to horizons 1–2 of wedge ice bodies at a depth of 15–20 m (*3*) and horizons 3–5 of relict massive ice at a depth of 30–60 m (*4*); *5*, *6* – power trendlines for mean ER values of high resistivity layers of the Ice Complex to depths of 20 m (*5*) and below (*6*).

[9] Fig. 7, *a* shows numbering of VES points, spaced at a distance of 1 km. Numbers of VES points spaced at 200 m are not indicated for the lack of space, but if necessary they can be found in the AR section (Fig. 3, *b*).

The almost total, dramatic, W–E trending degradation of relict massive ice at the 697-th km was preceded by the total ice content decreasing gradually over the study area, and sharply (to a depth of 20 m) at the 705-th km in the upper part of IC. These patterns were derived from the ER trends[10] in high-resistivity layers of the geoelectric section (Fig. 7, *c*), as well as from the DEMP data on subsurface to a depth of 10 m, i.e. to the part of IC associated with the occurrence of horizons 1 and 2 of wedge ice body (Fig. 4).

According to the landscape-geomorphological survey, geocryologists draw the southern border of the IC within the stretch of the projected railway line at the Maganskaya and Emilskaya terraces junction (704-th km), representing the end of the alas province of the Central Yakutia lowland of the right-bank Lena valley (Fig. 4).

The absence of alas suggests zero relict massive ice on the Emilskaya terrace, against the ice-rich Maganskaya terrace. This standpoint of geocryologists is based on putative thermokarst origin of alases in the wake of melted relict massive ice. However, the obtained VES data do not confirm the hypothesis in this particular case, showing that the disappearance of alases does not provide ground for postulating complete degradation of relict massive ice in the Ice Complex.

Despite the existing divergence in interpretation of permafrost and geophysical data, the difference in estimated positions of the boundaries delineating essentially changed (decreased) ice content at 705-th km, between the data obtained from DEMP, VES and landscape-geomorphological surveys proved to be 1 km. Such a small difference attests to the effectiveness of the geophysical methods applied to solving problems of permafrost engineering-geological zoning at different scales as part of applications of geological and geographical techniques within the Central Yakut Lowland area, with the Lena-Amga interfluve being its part, in the context of the region's ongoing development.

Elucidation on the structure and thickness of the IC southern margin of Lena-Amga interfluve within the study site marked as its intersection with projected railway line (the 694–707-th km stretch) will be considered by comparing the VES points cluster analysis results with the YAPSE drilling data and some fragmentary geocryological materials.

Firstly, we find it appropriate to highlight such common fact that the Lena-Amga interfluve is featured by the Ice Complex increasing in thickness against the background of Quaternary and Tertiary sediments showing a trend for their thinning in the transition area from low-altitude ice-rich Bestyakhskaya terraces to medium-altitude ice-rich – Tyungyulyunskaya, Abalakhskaya, Maganskaya – terraces on the righ-bank Lena valley.

Geocryologists often identify the IC of Lena-Amga interfluve with the position in the geological section of the lower boundary of a unified stratum of ice-rich deposits which include more or less uniform relict massive ice (not subdivided into horizons). Such a broad approach inferred from the classical definition of the Ice Complex given by P.A. Soloviev [*1959, p. 49*], was widely accepted in the last century and has survived to the present day. There has been relatively few estimated IC thicknesses in different locations of the Lena-Amga interfluve, with their overview provided below.

In the north of the Lena-Amga interfluve (upper reaches of the Tatta river), the IC thickness averages 25 m [*Ivanov and Katasonov, 1973*]. Within several physiographic areas (Tyungyulyun, Abalakh, cis-Suolye, cis-Tatta areas) in the adjacent northern margin of the interfluve, the IC thickness measures 15–30 and, more rarely, 40–50 m [*Ivanov, 1984*]. According to the detailed geological and geophysical studies carried out by the Melnikov Permafrost Institute staff back in 2005–2007 at the test site on the Maganskaya terrace along the AYAM mainline (the Tommot–Kerdyom corridor), the IC was found merging with the bedrocks (sandstones, limestones with clayey interlayers) and measured 28 m in thickness.

N.P. Bosikov [*1991, p. 40*] (without reference to any source) reported the IC thickness to vary betwen 10–50 and 30–60 m on the Tyungyulyunskaya and Abalakhskaya terraces.

According to P.A. Soloviev [*1959, pp. 22, 26, 28*], the IC thickness in the lacustrine-alluvial sequence of clay loams of the Abalakhskaya terrace varies from 15–20 to 40–60 m. Besides, P.A. Soloviev [*1959, p. 50*] provides general IC parameters within the entire Lena-Amga interfluve. These allowed to infer that the position of the upper boundary of the Ice Complex most often occurs at a depth of 1.5–3.0 m deflecting toward both smaller (1.0–1.2 m) and larger (4–5 m) side. Its lower boundary position is very variable, tending to be at a depth between 9 and 12 m.

The IC thickness is generally low and is approximately estimated at 5–10 m. However, the drilling data from some wells and alas basins incision depths suggest that it can reach 20–25 and 30–40 m. The statistics of the above data has demonstrated that the IC thickness distribution within the entire Lena-Amga interfluve agrees well with the probabilistic Gauss

[10] Average ER values in the studied area of the IC distribution have reduced by 4.6 times to a depth of 20 m, and by 2.7 times to a depth of 30–60 m.

law[11] with a dispersion of estimates in the range of 5–60 m averaging 30 ± 7.6 m at the 95 % confidence level.

By comparing the obtained data with the statistics of generalized VES points clusters with respect to the indicator of the median mean (the most resistant to random anomalous "outliers" at the extreme ends of the analyzed data set) it was found that the approximate estimates of the IC thickness provided in the openly published literature agree well with the occurrence depths of the IC lower boundary in the 3rd horizon of wedge ice body (Table 2).

Yet another comparison made with respect to the same indicator of the median average, which involved the VES points cluster analysis result and the YAPSE drilling data, allowed the inferences described below. The 1st horizon of wedge ice body occurring generally at a depth of 2–6 m, failed to have a pair in the clusters identified on the geoelectric section. However the 2nd and 3rd horizons of wedge ice body occurring in the 5–8 and 9–10 m depth intervals (the YAPSE drilling data) have pairs from the 3rd and 2nd clusters (1st and 2nd horizons of wedge ice body, occurring at a depth of 5–8 and 7–15 m, respectively). While the 2nd horizon of wedge ice body in the generalized representation not only occurs exactly at a depth of 5–8 m, but also almost equally rarely (with a probability of 18 and 32 %) appears in the cross-section according to YAPSE and VES data, showing a good agreement with the 3rd horizon. The position of its generalized boundaries in the IC is estimated at 8–10 and 7–15 m with a probability of 5 and 25 % inferred from the YAPSE drilling data and VES materials.

Within the studied southern margin of the Ice complex, either individual fragments or their integrated whole, as well as wedge ice bodies unfailingly rimmed with ice soils were detected by point drilling and VES volumetric sounding at different depths (probability of about 40 and 70 %).

Comparison of the materials resulting from mathematical processing of VES curves, clustering and geological interpretation of the geoelectric section parameters with general geological data obtained on the Lena-Amga interfluve, and specific engineering-geological data (available from YAPSE and MPI materials) thus allow to recognize VES results as reliable and advantageously applicable to the study of structure and thickness of wedge ice bodies both across the entire section, and in separate horizons.

CONCLUSION

The results of studies obtained by VES technique on the 694-707-th km stretch of the Berkakit–Tommot–Yakutsk railway indicate the previously unknown multi-horizon structure of massive ground ice in the southern margin of the Ice Complex of the Lena-Amga interfluve.

According to statistical cluster analysis of VES curves, the generalized structural framework of the Ice Complex consists of five horizons: the upper and middle parts of the IC are composed to a depth of 20–30 m by 1–3-m thick layers of wedge ice; the lower part of the IC is represented by the 4th and 5th horizons of relict massive ice of unknown (polygenetic) nature, occurring at a depth from 18–20 to 50–60 m.

Judging from the ER values of high-resistivity layers of the geoelectric section, the 3rd horizon of the IC occurring on average at a depth of 10–30 m is seen as the most saturated with massive ice (wedge ice).

The southern boundary of the IC shows evidence of almost completely degraded buried wedge ice (according to the VES method) at the 697-th km of the projected railway line. This location is marked by a deep hollow at the top of the Emilskaya terrace. Beyond its limits, wedge ice bodies occur sporadically to a depth of <10 m, having however irregular distribution (according to drilling and VES data) with a probability of occurrence of about 0.1 %.

It follows from the ER dynamics analysis of high-resistivity layers of the geoelectric section (VES and DEMP data), that the degradation of relict massive ice was preceded by a sharp decrease in ice content of the sediments at the 705-th km point of the railway. According to landscape-geomorphological zoning this location is a junction between Maganskaya (moderately high-standing, very ice-rich sediments) and the right-bank Lena valley Emilskaya (significantly lower ice content in sediments, high-standing) terraces, which is associated with the terminus of alas-valley province of the Lena-Amga interfluve in the Central Yakut Lowland.

The difference in estimates of the boundary position between geophysical data and landscape-geomorphological zoning is insignificant (about 1 km). The VES and DEMP methods are proposed to be used as part of geological and geographical methods for geological-engineering zoning and permafrost extent survey within the Central Yakut Lowland.

References

Abolin, R.I., 1929. Geobotany and Soils of the Lena-Vilyuy Plain. Izd-vo AN SSSR, Leningrad, 372 pp. (in Russian)

Bosikov, N.P., 1991. Evolution of Alases in Central Yakutia. IMZ SO AN SSSR, Yakutsk, 128 pp. (in Russian)

Frantov, G.S. (Ed.), 1984. Electrical Prospecting: A Practical Guide. Nedra, Leningrad, 534 pp. (in Russian)

[11] Interestingly, the range of probabilistic distribution of the average estimated thicknesses of the Lena-Amga Ice Complex in part includes a particular estimate of total IC thickness for South Chukotka. Here in the Main river valley, in the "Ledovy Obryv" outcrop, occurs the fourth horizon of wedge ice body with a total thickness up to 30 m closely approximating to the river water edge [*Kotov, 1988*].

Grave, N.A., 1944. Ground ice of the Lena and Aldan watershed. In: Trudy Instituta Merzlotovedeniya (Proc. of Obruchev Permafrost Research Institute). Moscow, vol. 4, pp. 10–32.

Grinenko, V.S., Kamaletdinov, V.A., Slastenov, Yu.L., Shcherbakov, O.I., 1995. Geological structure of the Bolshoy Yakutsk. In: Regional Geology of Yakutia. Yakutsk University Press, Yakutsk, pp. 3–20. (in Russian)

Ivanov, M.S., 1984. Cryogenic Structure of Quaternary Deposits of the Lena-Aldan depression. Nauka, Novosibirsk, 125 pp. (in Russian)

Ivanov, M.S., Katasonov, E.M., 1973. Cryogenic deposits of the upper reaches of the Tatta River (Central Yakutia). In: II Intern. Conf. on Permafrost. Papers and presentations. Yakutsk, vol. 2, pp. 106–109. (in Russian)

Kalinin, V.M., Yakupov, V.S., 1989. Regional Patterns of Permafrost Thickness Behavior. YaNTs SO AN SSSR, Yakutsk, 144 pp. (in Russian)

Khmelevsky, V.K., Shevnin, V.A. (Ed.), 1994. Electrical Prospecting using Resistivity Methods. Moscow University Press, Moscow, 246 pp. (in Russian)

Korkina, R.I., 1949. Report on electrometric studies in Yakutia. In: Unpublished Reports, Permafrost Institute AS USSR. Moscow, Cat. No. 454, 55 pp. (in Russian)

Kotov, A.N., 1988. Multi-facies complex of Late Pleistocene syn- and epicryogenic deposits of the Mayn River Valley (Chukchi Peninsula). In: Problems of Geocryology. Nauka, Moscow, pp. 108–115. (in Russian)

Kulaichev, A.P., 2006. Methods and Tools for Integrated Data Analysis. FORUM: INFRA-M, Moscow, 512 pp. (in Russian)

Melnikov, P.I., 1968. Permafrost conditions for construction in Yakutia. In: Building Foundations on Permafrost in Yakutia. Nauka, Moscow, pp. 5–19. (in Russian)

Melnikov, V.P., 1977. Electrophysical Investigations of Frozen Ground. Nauka, Novosibirsk, 108 pp. (in Russian)

Melnikov, V.P., Niskovskikh, N.S., 1978. On classification of permafrost-geological elements from their electrochemical activity. In: Geocryological and Hydrogeological Investigations in Yakutia. IMZ SO AN SSSR, Yakutsk, pp. 58–65. (in Russian)

Neradovskii, L.G., 2008. The study of Ice Complex at the Lena-Amga interfluve. Geoekologiya, Inzhenernaya Geologiya, Gidrogeologiya, Geokriologiya, No. 5, 460–467.

Soloviev, P.A., 1959. Permafrost in the Northern Part of the Lena-Amga Watershed. Izd-vo AN SSSR, Moscow, 144 pp. (in Russian)

Spektor, V.B., Spektor, V.V., Bakulina, N.T., 2011. Buried snow in the Lena-Amga Plain. Kriosfera Zemli [Earth's Cryosphere] XV (4), 16–20.

State Standard, 2018. Working Document GOST 25100-2011. Soils. Classification. Standartinform, Moscow, 38 pp. (in Russian)

Svetov, B.S., Berdichevskii, M.N., 1998. Electrical geophysical prospecting methods at the present stage. Geofizika, No. 2, 4–11.

User Manual, 2001. IPI2Win Software (ver. 2). Moscow , 22 pp. (in Russian)

Varlamov, S.P., Skryabin, P.N., Skachkov, Y.B., Shender, N.I., 2006. Geocryological conditions in icy permafrost terrain along the proposed Tommot-Kerdem Railway. Nauka i Obrazovanie, No. 4 (44), 78–83.

Yakubovsky, Yu.V., Lyakhov, L.L., 1988. Electrical Prospecting. Nedra, Moscow, 390 pp. (in Russian)

Zaborovskiy, A.I., 1963. Electrical Prospecting. Gostoptekhizdat, Moscow, 429 pp. (in Russian)

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