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PHYSICAL AND CHEMICAL PROCESSES IN FROZEN GROUND AND ICE

ESTIMATION OF THE FREEZING INTENSITY OF THE SALT WATER DROPS IN THE COURSE OF WINTER SPRINKLING

A.V. Sosnovsky, N.I. Osokin

Institute of Geography, Russian Academy of Sciences,
Staromonetniy lane 29, Moscow, 119017, Russia; alexandr_sosnovskiy@mail.ru

The simplified dependencies for estimation the fraction of the ice formed in a drop of fresh water when falling in the atmospheric air have been obtained. Based on mathematical modeling, the intensity of the freezing of salt-water droplets with various variants of salt rejection from the freezing boundary has been determined. An estimate of the increase in air temperature in a droplet plume consisting of drops of salt water has been given. The fraction of ice in a drop of salt water falling in atmospheric air and in a droplet plume has been calculated depending on the air temperature and the size of the drops.

Key words: *freezing drops, winter sprinkling, intensity of freezing-up, mathematical modeling, salt water.*

INTRODUCTION

In recent years, the winter sprinkler method has been used increasingly to freeze up ice. It is based on the use of long-range sprinkler systems to spray water and create a droplet plume. Such sprinklers are widely used in agriculture to irrigate crops. Based on the serial sprinkler DDN-70 (long-range mounted sprinkler with a jet range of 70 m), the Grad-1 sprinkler was developed for sprinkling in winter conditions [*Design..., 1991*]. When the sprinkler is operating, the stream of water rises to a height of 20 m, and then breaks up into small drops, which fall to the ground. The droplet diameter is generally 1–2 mm. The dimensions of the droplet plume (the volume of the space in which the drops fall) are 15–20 m in height, 30–40 m in length and 5–10 m in width. The size of the plume depends on the wind speed. With increasing wind speed, the height of the plume decreases and its width increases. The sprinkler can operate by moving both in a circle and in a sector or with a fixed position of the sprinkler barrel.

An ice shell forms on the surface of the freshwater drops falling in frosty air. When more than 55 % of the volume of a water droplet freezes, the ice shell (in that time the shell thickness is 0.23 of the droplet radius) does not break when it hits the underlying surface. Experiments on winter sprinkling in Yakutia [*Gordeichik, Sosnovsky, 1982*] have demonstrated that more than 60 % of the drop volume freezes at an ambient temperature below –50 °C. At the same time, dry granular ice is formed, consisting of partially or completely frozen water droplets, that ice is not suitable for the construction of an ice crossing.

When the ice shell of the drop breaks, unfrozen water flows out of the drop and a water-ice mixture forms on the surface of the earth. After its freezing, the monolithic ice with the density of 800–850 kg/m^3 is formed. At present, winter sprinkling is widely used for the construction of ice crossings and winter roads [*Design..., 1991*]. The foreign experience in creating artificial ice islands demonstrates that the ice freezing-up by means of sprinkling provides the best effect [*Kubyshkin et al., 2018*]. The method has a great advantage over others in the speed of creating large masses of frozen ice, and it is accepted as the main method for creating artificial ice islands in the international standard ISO 19906.

With continuous sprinkling and removal of unfrozen water outside the freezing zone, the porous ice with a density of 400–600 kg/m^3 is formed. With sprinkling during the day, the height of the porous ice massif can exceed seven meters [*Sosnovsky, Khodakov, 1995*]. The studies carried out to date have revealed that one of the most economical methods of desalination and purification of large volumes of saline water is the drip freezing-out method, which is implemented during winter sprinkling [*Sosnovsky, Khodakov, 1995; Gao et al., 2004; Biggar et al., 2005*]. So, with the mineralization of frozen water up to 10 g/L, the mineralization of porous ice does not exceed 1 g/L.

The freezing of salt water has a number of features. The freezing point of salt water and the temperature of its highest density depend on its salinity. The salinity of the frozen ice is several times lower than that of the original water. Those features entail differences in convection, the mechanism of ice formation, and the thermal regime of waters of different salinity during freezing.

One of the factors affecting the intensity of ice formation during winter sprinkling is the supercooling of water droplets. A detailed review of that problem is presented in [*Smorygin, 1988*]. The supercooling of water droplets is investigated to prevent the icing of aircrafts in flight and the icing of sea vessels [*Kulyakhtin, Tsarau, 2014; Alekseenko et al., 2016*], as well as for the assessment of the ice-cover deposits and the possibility of ice-rain formation [*Vilfand, Golubev, 2011; Smorodin et al., 2014*]. When studying the phenomenon of supercooling, the distilled water purified from impurities is used to obtain significant supercooling. When supercooling stops, ice grows rapidly in a drop of water. The possible mechanisms of branching of needle-shaped ice crystals in supercooled water have been experimentally investigated in the [*Shibkov et al., 2013*]. The growth of branched crystals also occurs when salt-water droplets freeze [*Adams et al., 1963*]. The supercooling of the fresh-water and salt-water droplets in the open air has not been studied enough. Thus, the dependence of the time and magnitude of supercooling on the size of the droplets, the rate of cooling and the salinity of water is not clear. However, it is possible to assess the effect of possible supercooling of water droplets on the intensity of ice formation.

In the [*Sosnovsky, 1993*], on the basis of experiments and mathematical modeling, an assessment of the effect of supercooling of fresh water drops on the intensity of ice formation during winter sprinkling is given. The results of calculations have demonstrated that at the air temperature of −20 °C, and a supercooling of −8…−6 °C, the productivity of ice formation decreases by 5 % for water droplets with a diameter of 1–2 mm. The experiments in the open air with the use of fresh water have revealed that the supercooling of water droplets is about −1…−3 °C. Unfrozen drops have not been observed in the droplet plume, which mainly consisted of water droplets 1–2 mm in diameter. Perhaps that is due to the presence of a large number of tiny ice crystals in the plume, which serve as crystallization centers for falling water drops. Therefore, at the air temperatures below −10…−15 °C, the effect of supercooling of water on the intensity of ice formation during winter sprinkling can be neglected.

The aim of the research is to assess the intensity of ice formation in a drop of salt water and in a droplet plume during winter sprinkling.

FREEZING OF FRESH-WATER DROPS

The freezing intensity of salt-water drops must be compared to that of fresh-water drops. Therefore, let us first consider the freezing of fresh-water drops. During operation of the Grad-1 sprinkler, the height of the droplet plume is 18 m at a wind speed of 5 m/s and a jet range of 70 m. The water discharge with a nozzle diameter of 55 mm is 240 m^3/h. The falling time of water droplets with a diameter of 1.5 mm from a height of 18 m is 3.3 seconds at a vertical drop speed of 5.4 m/s [*Mason, 1961*].

When fresh-water droplets freeze, an ice shell forms on their surface, being thickened during the droplet's fall, and reducing the radius of the liquid part.

In the [*Sosnovsky, 1980*], the problem of freezing of the water-drop falling in frosty air has been considered. As a result of solving the heat conduction equation with the Stefan condition at the freezing boundary (phase boundary), and the condition of heat transfer at the droplet boundary using the known criterion dependences to determine the heat and mass transfer coefficients of the falling water-drops, a dependence has been obtained for determining the freezing time of a fresh-water drop (τ) on the position of crystallization front (ξ) (origin of coordinate is at the center of the drop) in the form of:

$$\tau = \frac{264R^2}{M}\left[\frac{109}{3\mathrm{Nu}}\left(1-\frac{\xi^3}{R^3}\right)+M_1\right], \qquad (1)$$

where Nu = 2 + 17.2$R^{0.815}$ is the Nusselt number; R is the radius of the drop, mm; $M = T_0 - T_a + 2.3(4.8 - fe_a)$; T_0 = 273 K; T_a is an ambient air temperature, K; e_a – water vapor pressure, kg/m^3; f – air humidity, in fractions of unit; $M_1 = (1 - \xi^2/R^2)/2 - (1 - \xi^3/R^3)/3$.

The fraction of ice in a drop (P) is calculated from the dependence of P=1− ξ^3/R^3.Then equation (1) can be transformed to the form, taking into account that $\tau = h/v_f$,

$$P = \frac{3\mathrm{Nu}}{109}\left[\frac{hM}{264v_f R^2}+M_1(P)\right],$$

where h is the drop-fall height; v_f is the drop-fall rate.

When the proportion of ice in a water droplet is $P < 0.6$, the contribution of the second term on the right-hand side of M_1 to P does not exceed 2.4 %.

For express-assessments, the resulting dependence has been simplified as much as possible. Thus, to calculate the fraction of ice (P) in a drop of fresh-water with a diameter of d = 1–2 mm, a simplified dependence has been obtained in the form of [*Sosnovsky, 1983*]

$$P = h\,|t_a|/(500d^2)$$

and taking into account that $h = \tau v_f$, we obtain

$$P = \tau v_f |t_a|/(500d^2), \qquad (2)$$

where t_a is the air temperature, °C; d is the droplet diameter, mm.

The calculations using the formulas (1) and (2) with an air temperatures of above −10 °C coincide

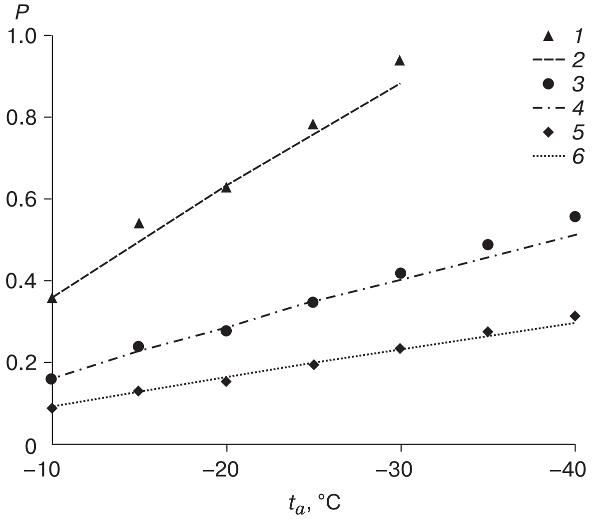

Fig. 1. Proportion of ice (P) in the drop of fresh water falling from a height of 18 m.

1, 3, 5 – calculation by the simplified formula (2); *2, 4, 6* – calculation by the model dependence (1). Drop radius: *1, 2* – 0.5 mm; *3, 4* – 0.75 mm; *5, 6* – 1 mm.

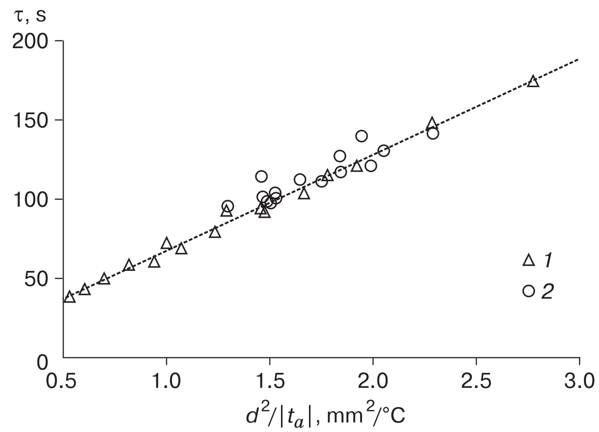

Fig. 2. Dependence of the time of complete freezing of fresh-water drops (τ) on the value of $d^2/|t_a|$:

1 – by the formula (2) for drops with a diameter of d = 3, 4 and 5 mm; *2* – by the data of the [*Balkarova et al., 2011*].

with an accuracy of 8 %. At lower air temperatures, the contribution of evaporation to heat transfer decreases; therefore, the calculations using the formula (2) overestimate the share of ice in a drop. To maintain an 8 % accuracy of calculations with an air temperature of −10...−40 °C, a correction factor is introduced: the results of calculations using the formula (2) are multiplied by 0.87. Because of that correction, the calculations by the formula (2) of the ice fraction for droplets with a diameter of 1–2 mm coincide with an accuracy of 8 % with the results of calculations by the formula (1) within the air temperature range of from −10 to −40 °C (Fig. 1).

The processing of the experimental data of the [*Mason, 1961*], carried out by the authors, has revealed that the droplet falling can be taken as $v_f = 6.42R^{0.63}$ m/s, where R is the radius in mm. In that case, the approximation confidence factor is $R^2 = 0.995$.

Let us compare the calculations by the formula (2) with the experimental data of other authors. In the [*Balkarova et al., 2011*], the results of measuring the time of complete freezing of fresh water droplets with a diameter of d = 3.8–5.2 mm with an air temperature t_a from −8 to −17.2 °C, floating freely in the air flow, are presented. The dependence of the time of complete freezing of a drop of fresh water τ according to the data of that work on the value of $d^2/|t_a|$ as well as the calculations by formula (2) of the value of τ at $P = 1$ (the proportion of ice in a drop of water when it is completely frozen) can be seen in Figure 2. It follows from Fig. 2 that the results of calculations of the time of complete freezing of a water drop according to the theoretical formula (2) and the measurements presented in the [*Balkarova et al., 2011*] have a good match.

FREEZING OF THE SALT-WATER DROPS IN THE COURSE OF WINTER SPRINKLING

The freezing of salt water occurs as a result of the selective growth of ice crystals, accompanied by the formation of cells and interbeds of brine between them [*Adams et al., 1963*]. As the temperature drops, new fresh-ice crystals fall out of the brine. Therefore, the salinity of the brine increases until a state of thermodynamic equilibrium is established at a given temperature. Thus, a certain phase composition corresponds to each value of the temperature of the ice formed from salt water.

When the drops of fresh water or slightly mineralized water (up to 10 g/L) freeze, an ice shell forms on the surface of the drop and thickens, reducing the radius of the liquid part. When water droplets with a higher salinity freeze, the branched crystals can grow deep into the liquid part of the droplet and penetrate into the brine. However, it is difficult to predict the growth of branched crystals. Moreover, as the drop freezes, the volume of its liquid part decreases, its mineralization increases, and its freezing temperature decreases. The amount of brine in the ice shell decreases as the ice cools because of the advance of the crystallization front.

The phase composition of salty ice depends on temperature and salinity. With a salt ice temperature of above −8...−10 °C, in the first approximation, a linear relationship between the concentration of brine S_b (kg/m^3) in ice and the ice temperature t_i can be assumed [*Doronin, 1969*]:

$$S_b = \sigma t_i, \qquad (3)$$

where $t_i = T_i - 273$; T_i is the ice temperature, K; the values of the σ coefficient depend on the freezing point of various salts, for sea water $\sigma = -18.2$ kg/(m^3·K). That dependence can be used to estimate the temperature of the beginning of freezing of the brine. As the salinity of the brine S_b increases, its freezing temperature t_i decreases in accordance with the formula (3).

Thermophysical calculations take into account the effective heat capacity of salted ice c_{ie}, which is equal to the weighted average value of the sum of the heat capacities of the crystalline ice c_i and the brine c_b, with taking into account the melting heat of ice L. The value of c_{ie} is determined by the formula [*Doronin, 1969*], with taking into account the salinity of ice S_i

$$c_{ie}\left(T_i\right) = c_i + \left(c_b - c_i\right)\frac{S_i}{\sigma\left(T_i - 273\right)} - \frac{LS_i}{\sigma\left(T_i - 273\right)^2}. \qquad (4)$$

The problem of determining the dynamics of freezing of a drop of salt water is the uncertainty at the phase boundary, since it is not clear what part of the salts is rejected into the liquid part of the drop, lowering the freezing point of salt water at the phase front. Therefore, the extreme variants of the rejection of salt ions from the freezing boundary into the liquid part of the drop have been chosen:

1 – salt ions are not rejected into the liquid part of the drop, the salinity of the ice shell and of the liquid core are equal, while the effective heat capacity of ice is determined by formula (4);

2 – salt ions during the freezing are completely rejected into the liquid part of the drop, while the heat capacity of the ice shell is equal to the heat capacity of the crystalline ice c_i; the salinity of the liquid part of the drop increases with the increasing of the thickness of the ice shell of the drop, and the temperature of the onset of freezing (the phase transition temperature t_i) is recalculated at the known salinity of the brine (a liquid part of the drop) according to the formula (3).

For the first option, the empirical formula of V.L. Tsurikov determining the salinity of the sea ice depending on the rate of its growth can serve as some justification [*Nazintsev, Panov, 2000*]:

$$S_i/S_w = 7w^{0.5}/(7w^{0.5} + 10.3),$$

where S_i is the ice salinity, ‰; S_w is the salinity of sea water, ‰; w is the rate of ice growth, mm/h.

At a high growth rate of salted ice up to 20 mm/h (at low negative air temperatures and significant wind), the salinity of ice according to V.L. Tsurikov will be 75 % of the initial one. When a drop with a diameter of 1.5 mm freezes, even at a small negative air temperature of −10 °C, the freezing rate is 55 mm/h. In that case $S_i/S_w = 0.83$. At lower air temperatures the S_i/S_w rises to 0.9.

The rationale for the second option is as follows. The moisture content of the porous ice frozen from fresh water in the first few days is 10–12 %, and the water is located on the surface of ice crystals in the form of film moisture [*Sosnovsky, Khodakov, 1995*]. The experiments of the authors have demonstrated that when using the water with a salinity of up to 10 g/L, the salinity of the formed porous ice decreases by an order of magnitude, which corresponds to the preservation of about 10 % of unfrozen water in porous ice. Since part of the water remains on the surface of ice crystals, it can be assumed that when salt water freezes, almost all brine will also be concentrated in the film moisture on the surface of ice crystals, and the presence of brine cells inside the ice is insignificant. Such a scenario can occur if during the formation of ice shells of droplets, the capture of salt ions by growing ice crystals is insignificant, and most of them are rejected into the central unfrozen part of the droplet and flow out of the mass of porous ice when the ice shell breaks after falling.

The first option solves the problem of ice formation within the entire range of negative temperatures without isolating a freezing front. The problem is entirely due to the dynamics of heat exchange between a drop of water and the surrounding air, to the dependence of the effective thermal conductivity of salted ice and the dependence of the content of unfrozen water on temperature. When calculating the freezing of seawater in the [*Bogorodsky et al., 2009*], an extended section of the ice-unfrozen liquid mixture – a two-phase zone – is used, in the volume of which a phase transition occurs, and the liquidus condition is used (the line of complete melting of solid phases in the phase diagrams, above which there is only liquid). For an object of millimeter dimensions, it is advisable to apply the heat balance model for the entire volume of a drop, considered in the [*Sosnovsky, 1988*]. In that case, the change in the heat content of the drop

$$dQ_1 = -c_{ie}(T_i)\rho_w V_d\, dT_i$$

is determined by the heat flux at the drop boundary

$$dQ_2 = \alpha_{eff}\,(T_i(\tau) - T_{eff})F_d\, d\tau_s,$$

where α_{eff} and T_{eff} are the effective heat transfer coefficient and the reduced air temperature, respectively; F_d is the surface area of the drop; T_i is the temperature of the freezing drop; ρ_w and V_d are the water density and drop volume, respectively.

Equating dQ_1 and dQ_2, and solving the differential equation to determine the freezing time of salt water (τ_s), we obtain the following dependences:

$$\tau_s = -\frac{\rho_w V_d L}{\alpha_{eff} t_{eff} F_d}\left(f_i + \Delta f_i + A_i + A_b\right), \qquad (5)$$

where

$$\Delta f_i = \frac{t_{i0}}{t_{eff}} \ln t_1, \; A_i = \frac{c_i t_{eff}}{L}\left(\Delta f_i - \ln\frac{1 - t_{i0} t_{eff}^{-1}}{1 - t_i t_{eff}^{-1}}\right),$$

$$A_b = \frac{-c_b t_{i0}}{L} \ln t_1, \; t_1 = \frac{1 - t_{eff} t_{i0}^{-1}}{1 - t_{eff} t_i^{-1}}, \; t_{eff} = T_{eff} - T_0,$$

where $t_i = T_{i0} - T_0$; T_{i0} is freezing point of salt water, K; $T_0 = 273$ K; $S_i = \sigma t_{i0}$; $f_i = 1 - t_{i0}/t_i$.

For water droplets 0.5–4.0 mm in diameter, the dependences can be used to determine α_{eff} (W/(m^2·K)) and T_{eff} (K) [*Sosnovsky, 1988*]:

$$\alpha_{eff} = 44.8 R^{-0.3},$$

$$T_{eff} = 61.06 \cdot 10^{-2} [T_a + 2.325(70.08 + 10^3 f_a e(T_a)],$$

where T_a, f_a are the temperature and relative humidity of air; $e(T_a)$ is density of saturated water vapor (kg/m^3) at air temperature T_a (K).

Since droplets with a radius $R \leqslant 1$ mm can be considered spherical, then $V_d/F_d = R/3$. The A_i and A_b values demonstrate the contribution to the intensity of freezing-up of the heat capacity of ice crystals and brine. The dependence of the heat capacity of fresh ice (c_i) and brine (c_b) on temperature and salinity is determined by the following empirical expressions [*Doronin, 1978*]:

$$c_i = 2.12 + 0.0078 t_i, \text{ kJ/(kg·K)},$$

$$c_b = 4.19 - 4.55 \cdot 10^{-3} S_b, \text{ kJ/(kg·K)}.$$

With the ice temperatures above −10 °C, the value of c_i can be taken equal to 2.08 kJ/(kg·K) with an accuracy of 2 %, while for $c_b = 3.73$ kJ/(kg·K) within the range of salinity of brine from 35 to 200 kg/m^3, the error is no more than 13 %. The authors' calculations reveal that taking into account the contribution of the values A_i and A_b to the intensity of freezing-up, the error in the application of the considered values of c_i and c_b does not exceed 0.2 and 0.4 %, respectively.

Let us assume that in the process of freezing of a water drop with an initial salinity S_0 at the time τ, the ice fraction will be $f_i(\tau)$. Then, taking into account that the precipitation of crystals of the main components of salts occurs at lower temperatures, we obtain the brine concentration $S_b = S_0/(1 - f_i)$ and $f_i = 1 - S_0/S_b$. Taking into account the equitation (3), we obtain $f_i = 1 - f_b$, where $f_b = S_0/(\sigma t_i)$. Thus, we get a relationship between the proportion of ice in a drop of water and its temperature.

The results of calculations of the ice fraction (f_i) in a freezing water droplet with a diameter of 1.5 mm depending on the fall time (τ) according to the first option (with the initial water mineralization of 35 g/L and the air temperatures of −10 °C, −20 °C and −40 °C) are demonstrated in Fig. 3. The time of falling of a water drop 1.5 mm in diameter from a height of 18 m is 3.3 s. At a water salinity of 35 g/L and the air temperature of −20 °C the water drops with a diameter of 1.5 mm, falling from a height of 18 m, will freeze by 27 % of their volume (Fig. 3). Whereas if the water is fresh, under the same initial conditions, the proportion of frozen water in the drop will increase by 11 %, reaching 30 % of its volume (Fig. 1). In that case, the temperature of the freezing drop of salt water will be −2.7 °C. At an air temperature of −10...−30 °C, the temperature of the drop of salt water falling from a height of 18 m will be −2.3 and −3.5 °C at the end of the fall, respectively.

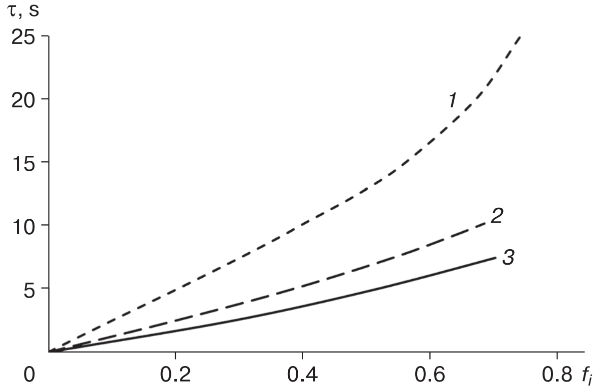

Fig. 3. Dependence of the proportion of ice (f_i) in a freezing drop on the time of falling (τ) according to the first option with the mineralization of the initial water of 35 g/L and air temperature of −10 °C (*1*), −20 °C (*2*), −40 °C (*3*).

Let us consider the second option, i.e. the complete rejection of salt ions from the freezing boundary into the liquid part of the drop. For mathematical modeling and calculations, we will accept (as in the case of a fresh water drop) a freezing scheme with the formation of an ice shell symmetrical advancing towards the center of the drop. The modeling assumes that the initial droplet temperature is equal to the freezing point at the initial salinity. In that case, the internal circulation of liquid in a freezing drop is not considered [*Sultana et al., 2017*].

In the presence of a phase boundary in a freezing water drop, the heat transfer and the freezing of water are described by the boundary-value problem of thermal conductivity provided that in the ice shell $\xi(\tau) < r < R$ and in the liquid part of the drop $0 < r < \xi(\tau)$, where r is the coordinate along the drop's radius:

$$\frac{\partial T_i}{\partial \tau} = a_i \left(\frac{\partial^2 T_i}{\partial r^2} + \frac{2}{r}\frac{\partial T_i}{\partial r}\right), \; \xi(\tau) < r < R,$$

$$\frac{\partial T_w}{\partial \tau} = a_w \left(\frac{\partial^2 T_w}{\partial r^2} + \frac{2}{r}\frac{\partial T_w}{\partial r}\right), \; 0 < r < \xi(\tau), \qquad (6)$$

where T_w is the temperature of the liquid central part of the water drop.

The heat transfer condition on the droplet surface is

$$-\lambda_i \frac{\partial T_i}{\partial r}\bigg|_{r=R} = \alpha_{eff}\left(T_i(R) - T_{eff}\right). \quad (7)$$

Stefan's condition at the freezing boundary provided $r = \xi(\tau)$ is

$$\lambda_i \frac{\partial T_i}{\partial r} - \lambda_w \frac{\partial T_w}{\partial r} = -L\rho_i k_i \frac{d\xi}{d\tau}. \quad (8)$$

At the freezing boundary, the temperature equality condition of the liquid and solid parts of the drop is applied.

In the central part of the drop with $r = 0$, the following condition is assumed:

$$\frac{\partial T_w}{\partial r} = 0\,. \quad (9)$$

In the initial period, the water temperature is taken equal to the freezing point of salt water (T_0): $T_w = T_0$, $\xi = R$ at $\tau = 0$.

Here the following designations are accepted: $a_i = \lambda_i/(c_i\rho_i)$ is the thermal diffusivity coefficient of ice, m^2/s; $a_w = \lambda_w/(c_w\rho_w)$ is the thermal diffusivity coefficient of water, m^2/s; λ_i is the thermal conductivity coefficient of ice, W/(m·K); c_i is the heat capacity of ice, kJ/(kg·K); ρ_i is ice density, kg/m^3; λ_w is the thermal conductivity coefficient of water, W/(m·K); c_w is the heat capacity of water, kJ/(kg·K); ρ_w is the density of water, kg/m^3; L is the heat of melting of ice, kJ/kg; k_i is the coefficient of influence, fraction of unit.

The k_i coefficient takes into account the influence of the spherical surface of the phase boundary on the freezing time of the next layer. When a water drop freezes, the time required to move the phase front to one spatial node of the computational grid decreases as the front shrinks to the center. Heat losses for the phase transition from a layer with radius ξ to a layer with radius $(\xi - \Delta\xi)$ are $(4/3)\times \times\pi\rho L(\xi^3 - (\xi - \Delta\xi)^3)$, where $\Delta\xi$ is the spacing of the spatial grid. The value of the heat flux through the phase surface during the time $\Delta\tau$ is equal to $4\pi\xi^2 Q_\xi \Delta\tau$, where Q_ξ is the left side of equation (8). Equating the heat loss to the value of the heat flux and neglecting the value of $\Delta\xi^3$, we obtain $k_i = 1 - \Delta\xi/\xi$.

INTENSITY OF WATER-DROP FREEZING

In the [*Sosnovsky, Glazovsky, 2018*], when solving the system of equations (6)–(9) and calculating the freezing intensity of a drop of salt water, the different variants of the growth of mineralization of the unfrozen part of the drop have been considered. Calculations have shown that the freezing time of a drop of salt water increases in comparison with the time of freezing of a drop of fresh water. That difference grows with an increase of the frozen part of the droplet and is most significant at low negative air temperatures. The freezing time of the droplet's half-volume for drops with the diameter of 1.5 mm (the water salinity is 35 g/L and the air temperature is −10 °C) is 25 % longer than that for fresh water, and 17 % longer at the air temperature of −20 °C. It has been assumed that 1/3 of the salt is retained in the ice shell of the drop when it freezes.

The system of equations (6)–(9) is solved for the water with a salinity of 35 g/L and the freezing point of −1.8 °C, providing the completely rejection of the salt ions from the freezing boundary into the liquid part of the drop and the formation of the fresh-ice shell. The droplet diameters of 1.0 mm, 1.5 mm, and 2.0 mm are taken for calculations. The intensity of water-drop freezing depends on the magnitude of the temperature difference between the air and the surface of the ice shell. According to the calculations by second option, the surface temperature of a drop of 1.5 mm in diameter is −3.8 °C at the moment when the half-volume of the droplet have been frozen. With complete rejection of salt from the freezing boundary, the salinity of the liquid core reaches 70 g/L.

An important parameter for heat transfer between air and a drop of water falling in a droplet plume is the time-weighted mean of the droplet surface temperature. As a result of solving the system of equations (6)–(9), the dependence of the temperature of droplet surface on the dynamics of droplet freezing has been calculated. Thus, at the moment when the half-volume of the drop with a radius of 1.5 mm has been frozen up, the time-weighted mean surface temperature is −2.8 °C for an air temperature of −20 °C. At that, the surface temperature depends mainly on the percentage of the frozen droplet volume and, to a lesser extent, on the air temperature. So, for a drop with a diameter of 1.5 mm the surface temperature at the moment when half-volume of the droplet has been frozen up (according to the second option of calculation) is −3.7, −3.9 and −4.1 °C for the air temperatures −10, −20 and −40 °C, respectively.

The results of calculating of the freezing time of a water drop 1.5 mm in diameter depending on the ice percentage according to the second option for different air temperatures are demonstrated in Fig. 4. During the fall of a drop from a height of 18 m, 0.14, 0.20, 0.24 and 0.39 of the droplet's volume has time to freeze up at the air temperatures of −10, −15, −20 and −40 °C, respectively.

The calculations by the formula (1) have revealed that for the air temperature of −20 °C, it takes 5.6 seconds to freeze the half-volume of a drop of fresh water with a diameter of 1.5 mm. For salt water, according to the 1st option of calculations, the freezing of the droplet's half-volume takes 21 % more time,

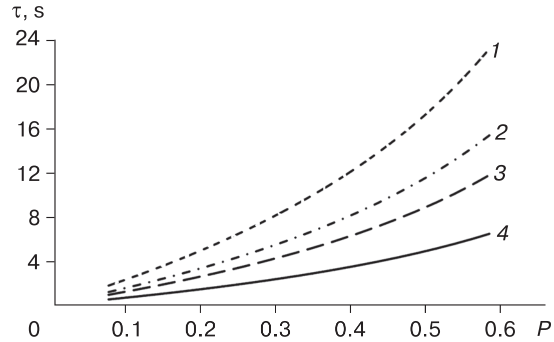

Fig. 4. Dependence of the freezing time of a drop with a diameter of 1.5 mm (τ) on the volume proportion of ice (*P*) with complete rejection of salt for air temperature of –10 °C (*1*), –15 °C (*2*), –20 °C (*3*), –40 °C (*4*).

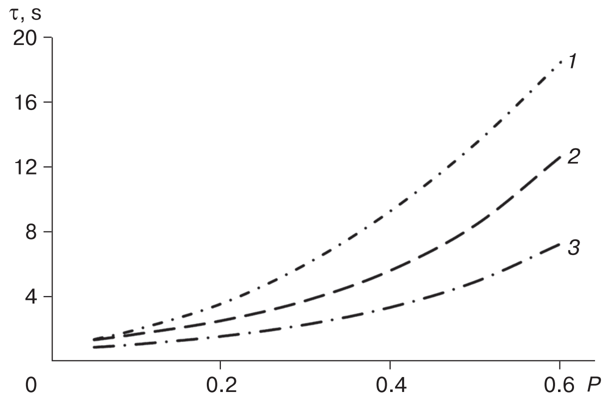

Fig. 5. Dependence of the freezing time of a water drop (τ) on the proportion of ice volume (*P*) according to the second option of calculations for an air temperature of –20 °C for drops with a diameter of 2.0 mm (*1*), 1.5 mm (*2*), 1.0 mm (*3*).

i.e. 6.8 seconds (Fig. 3); and according to the 2nd option, the same takes 8.4 seconds (Fig. 4). During the fall of a drop from a height of 18 m for an air temperature of –20 °C, 24 % of the ice volume freezes according to the 2nd variant of calculations (Fig. 4). That is 8 % less than by the 1st option of calculation and 25 % less than for fresh water.

For low negative air temperatures in order to increase the freezing-up performance the droplet size is reduced by using the nozzles of a smaller diameter. The influence of the droplet's size (diameters of drops are 1.0, 1.5 and 2.0 mm) on the intensity of freezing-up according to the 2nd option of calculations is demonstrated in Fig. 5. It reveals that for an air temperature of –20 °C, the freezing time of the droplet's half-volume according to the second variant of calculations are 4.9, 8.4 and 13.4 seconds for drops with diameters of 1.0, 1.5 and 2.0 mm, respectively.

HEAT EXCHANGE IN DROPLET PLUME

In the droplet plume, the air is heated due to the heat exchange of the falling water drop with the air and the release of the latent heat of ice formation. The intensity of heat release depends, among other things, on the temperature difference between the surface of the freezing water drop and the air in the plume. With an increase in the salinity of the initial water and an increase in the proportion of ice in the drop, the drop temperature and, as a consequence, the heat exchange rate decreases, reducing as a result the heating of air in the plume. The authors' calculations have demonstrated that during the fall of a drop (1.5 mm in diameter) its average temperature is –2.0 and –2.7 °C for salt frozen water (salinity of 35 g/L) and about –0.3 and –0.4 °C for fresh water at the air temperature of –10 and –40 °C, respectively.

For an assessment of the increase in air temperature in the droplet plume, in the [*Sosnovsky, 1983*], the dependence has been obtained in the form of:

$$\Delta T = \frac{T_i - T_a}{1 + 0.0121 v_1 R^{1.93} S_l G^{-1}}, \qquad (10)$$

where T_a is the temperature of the atmospheric air, K; T_i is the temperature of ice in a drop, K; v_1 is the speed of ventilation of the plume by the wind (the speed of blowing-off a drop of water by the wind is the difference between the speed of the wind and the horizontal speed of movement of the drop under the influence of wind force), m/s; R is the radius of drops, mm; S_l is plume length, m; G is water flow rate of the sprinkler, m^3/s.

The ventilation speed v_1 of a drop with a diameter of d = 1.5 mm is about 1.3 m/s for a wind speed of 5 m/s [*Sosnovsky, 1983*]. For $G/S_l = 18 \cdot 10^{-4}$ m^2/s (G = 0.065 m^3/s, S_l = 40 m), using the formula (10), we obtain the air temperatures of –8.7 and –33.9 °C for the ambient air temperature of –10 and –40 °C, respectively, in the plume for the water droplets with a diameter of 1.5 mm and a salinity of 35 g/L. For fresh water, the air temperatures in the plume are –8.3 and –33.6 °C for the ambient air temperature of –10 and –40 °C, respectively. As a result, the temperature of falling-in-air water droplets are higher than the temperature of the ambient air, and the proportion of the ice frozen-up in the plume is lower. The air temperatures in the droplet plume of fresh water differ slightly from those of salt one. Therefore, the main difference between the freezing of fresh water and salt one in the droplet plume is due to the peculiarities of salt-water-drops freezing.

To calculate the productivity of freezing salt water during winter sprinkling, we use the formula for the volumetric freezing of a drop (5) and the formula (10) for calculating the air temperature in a droplet

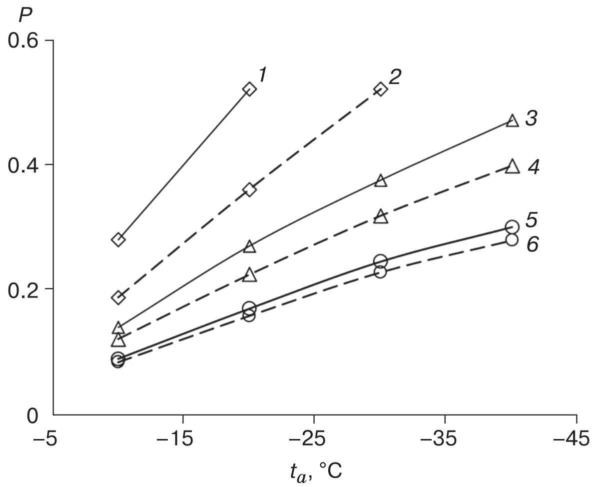

Fig. 6. Dependence of the proportion of ice (*P*) on the temperature (*1*, *3*, *5*) in a drop of water with a salinity of 35 g/L and in a droplet plume (*2*, *4*, *6*) 18 m high for drops with a diameter of:

1, *2* – 1.0 mm; *3*, *4* – 1.5 mm; *5*, *6* – 2.0 mm.

plume. The results of calculations of the dependence of the ice proportion in the water droplet with a salinity of 35 g/L and a diameter of 1.0, 1.5, and 2.0 mm as well as in the droplet plume on the air temperature are demonstrated in Fig. 6. For water droplets with a diameter of 1.5 mm, the difference in the ice proportion in an individual droplet and in a droplet plume is 15–18 %, whereas for the drops with a diameter of 2 mm, that difference is 6–8 %, and for those with a diameter of 1 mm, it is more than 44 %.

CONCLUSION

An assessment of the intensity of freezing of salt-water drops during winter sprinkling has been accomplished . A simplified relationship which makes it possible to estimate the proportion of ice in a drop of fresh water for an ambient temperature below –10 °C has been obtained. According to the data of independent studies, the model of freezing of fresh-water drops has been verified. The freezing of water droplets with increased mineralization, in particular, sea-water droplets, has been considered. When salt-water droplets freeze during the rapid ice formation process, both the growth of branched crystals permeating the entire volume of the droplet and, possibly, the formation of a phase front (as in a drop of fresh water or slightly mineralized one) can occur.

When solving the phase problem of freezing of a salt-water drop, the main problem is the uncertainty in determining the amount of the salts captured by the growing ice, and accordingly, in the increase of the salinity of the liquid part of the drop. To assess the influence of that uncertainty, the extreme variants of salt rejection are considered: 1) salt ions are not rejected into the liquid part and their uniform distribution over the droplet volume is maintained; 2) salt ions are completely rejected into the liquid part of the freezing droplet. At that, different models of freezing of a drop of water were applied: in the first case, it was a previously developed model of volumetric freezing, in the second case, the front problem of freezing of a drop of salt water was solved with the Stefan condition at the phase boundary. After a series of calculations, for the first time, a comparison of the intensity of ice formation has been made for the considered freezing models. The results of calculations for an air temperature of –20 °C have revealed that according to those models the difference in the proportion of ice in a drop of salt water is about 8 %. Due to the slight difference in the intensity of freezing-up of a drop of water, it is advisable to use a simpler scenario of volumetric freezing for the calculations.

For the first time, the calculations of the increase in air temperature in a droplet plume of salt water are given. On their basis, the proportion of ice in a drop of salty water falling in the atmospheric air and in the droplet plume has been determined, depending on the air temperature and the size of the drops. Calculations have shown that for water droplets with a diameter of 1.5 mm, the difference in the proportion of ice in an individual droplet and in a droplet plume is 15–18 %.

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