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USE OF THE ANALYTICAL SOLUTION OF FUNCTIONING OF THE HORIZONTAL EVAPORATOR TUBULAR (HET) THERMOSIPHON SYSTEM FOR QUICK EVALUATION OF THE EFFICIENCY OF ITS WORK

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This paper presents the analytical model of the functioning of the system of temperature stabilization of soils of the horizontal evaporator tubular (HET) type based on the integral method. The solutions of numerical and analytical models for temperature stabilization systems of soils of the HET type with different lengths of the evaporating part, as well as for the Arctic cities with different climates–Salekhard, Varandey, and Igarka. A comparison of the results obtained within the framework of numerical and analytical solutions indicates that the developed analytical model can be used for a quick assessment of the functioning of the HET system of temperature stabilization of soils for various design solutions and climatic characteristics.

Keywords: *permafrost, soil, seasonal cooling device, HET system, condenser, pipeline, evaporator.*

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INTRODUCTION

Preservation of soils in a frozen state is an acute problem of construction works in permafrost regions. For objects with a relatively low heat release, heat-insulating coatings are sufficiently efficient , whereas for objects with intense heat release, it is necessary to use methods of active thermal stabilization of soils.

Among the currently available seasonal cooling devices, including both single devices and large collector systems with an increased depth of the evaporative part, a special place is occupied by the soil temperature stabilization system with a horizontal evaporator manufactured by NPO Fundamentstroiarkos LLC – the HET system (horizontal evaporator tubular system).

The works [*Anikin, 2009; Dolgikh et al., 2014*] are devoted to the development of a mathematical model of the functioning of this system and the numerical solution of the equations obtained. Thermal loads limiting the functioning of the system have been studied and analyzed [*Melnikov et al., 2017*], and optimal configurations of the system for various operating conditions have been determined [*Ishkov et al., 2019; Ishkov, Anikin, 2020*].

However, in order to determine all the parameters and assess the operating conditions of the HET system, it is necessary to use a complex numerical model and special software [*Anikin et al., 2017*], whereas for an ordinary engineer, the question of the efficiency of the system in the format "will the system cope with the thermal pressure from the structure under construction or not" is more interesting.

Therefore, the purpose of this article is to develop an analytical model that can be used for quick evaluation of the functioning of soil temperature stabilization system with a horizontal evaporator for various design solutions and climatic characteristics.

ANALYTICAL MODEL OF THE HET SYSTEM FUNCTIONING

Let us consider the general view of the HET system and the way it is installed. The scheme of the system is shown in Fig. 1.

The HET system is a steel structure with three functional units: an evaporator, a condenser, and a circulation accelerator. The evaporator is made in the form of a curved structure with 90°–180° rotations. The condenser has a developed fin surface with an area of about 100 m^2. The circulation accelerator is a pipe of a larger diameter than that of the evaporator, in which refrigerant vapors are separated from suspended droplets due to gravity. HET systems are

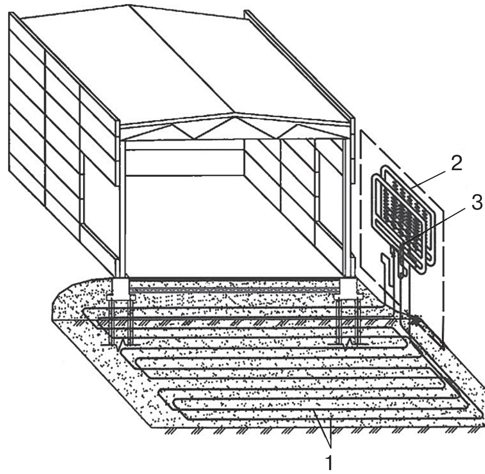

Fig. 1. Horizontal naturally acting evaporator tubular system (HET):

1 – evaporator; 2 – condenser; 3 – circulation accelerator (separator).

widely applied in the construction of structures on a pile foundation, with floors on the ground, as well as tanks with a base on a filling, etc.

The main advantage of this system is the ability to freeze large areas (and volumes) of soil under structures built on permafrost. In particular, for regions with virtually no winds (for example, Yakutia [*Pavlov, 2003*]), it is possible to install industrial refrigerators on a condenser fin grid. This increases the heat exchange between the condenser and the atmosphere, which, in turn, increases the efficiency of the HET systems.

To simplify the modeling of the soil freezing process, the HET system can be represented as straight pipe sections, the configuration of which is specified by their diameter and the distance between them (system laying step). Closing of soil freezing halos between the pipes of the HET system means the creation of a frozen soil stratum (volume) of high bearing capacity.

Let there be frozen soil around a pipe with radius b and length L, which is a cylinder with radius R_0 (Fig. 2).

If the boundary of the frozen soil moves much slower than the temperature inside the cylinder is set, a stationary solution can be considered for the temperature distribution:

$$\frac{1}{r}\frac{d\left(r\,dt(r)/dr\right)}{dr}=0, \tag{1}$$

where r is the radial coordinate, m; $t(r)$ is the temperature, °C. The solution of Eq. (1) is written as

$$t(r)=C\ln(r)+C_1, \tag{2}$$

where C, C_1 are constants to be determined.

The boundary conditions for the problem under consideration are written as

$$\begin{aligned} t(b)&=t_{ev},\\ t(R_0)&=t_{bf}, \end{aligned} \tag{3}$$

where t_{ev} is the temperature of the evaporator pipe, °C; t_{bf} is the temperature of the phase transition, °C.

From Eqs. (2) and (3), we get

$$t_{bf}-t_{ev}=C\ln\left(\frac{R_0}{b}\right)\ \rightarrow\ C=\frac{t_{bf}-t_{ev}}{\ln(R_0/b)}.$$

The heat flux dU, which is supplied to the element of the evaporation pipe of length dL, is equal in absolute value

$$dU=\lambda\frac{\partial t}{\partial r}2\pi r dL=2\pi\lambda\frac{t_{bf}-t_{ev}}{\ln(R_0/b)}dL, \tag{4}$$

where λ is the coefficient of thermal conductivity of frozen soil, W/(m·°C).

The total thermal power of the cooling system (U_{tot}) is limited by the efficiency of the condenser part ($\alpha S\eta$):

$$U_{tot}=\alpha S\eta\left(t_{con}-t_a\right). \tag{5}$$

Here U_{tot} is the total thermal power of the cooling system, W; α is the heat transfer coefficient of the condenser fins, W/(m^2·°C); S is the total surface area of the condenser fins, m^2; η is the heat transfer efficiency coefficient of the condenser fins (calculated to be 0.90); t_{con} is the temperature of the condenser fins, °C; t_a is the air temperature, °C.

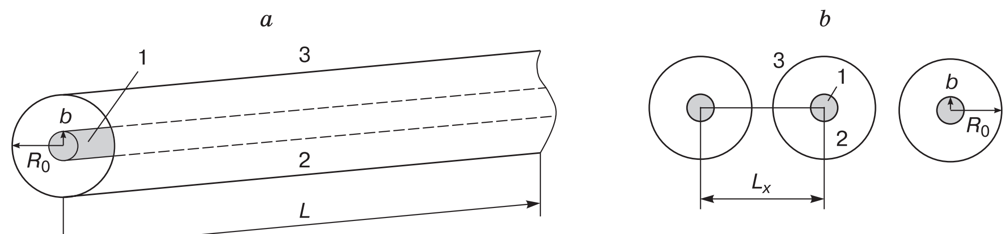

Fig. 2. View of the evaporator pipe from the side (*a*) and from the end (*b*).

1 – evaporator; 2 – frozen soil; 3 – unfrozen soil. L is the pipe length, m; L_x is the distance between the axes of the pipes of the evaporative system, m; b is the pipe radius, m; R_0 is the radius of the frozen soil cylinder, m.

As follows from the work [*Ishkov et al., 2019*], the temperatures of the condenser and the evaporator are related by the ratio

$$t_e = t_c + \frac{\rho_L g H}{dP/dt}, \tag{6}$$

where ρ_L is the density of the liquid refrigerant, kg/m^3; g is the acceleration of gravity, m/s^2; dP/dt is the dependence of the saturated vapor pressure on temperature, Pa; H is the height difference between the liquid level in the condenser and the considered point of the evaporator, m.

In turn, H is equal to

$$H = H_0 + L \sin \varphi, \tag{7}$$

where φ is the angle between the evaporator pipe and the ground surface.

Taking into account Eqs. (5)–(7), we have

$$t_{ev} = t_a + \frac{U_{tot}}{\alpha S \eta} + \frac{\rho_L g (H_0 + L \sin\varphi)}{dP/dt}. \tag{8}$$

Substituting (8) into (4), we get

$$dU = 2\pi\lambda \frac{\left(t_{bf} - t_a - \frac{U_{tot}}{\alpha S \eta} - \frac{\rho_L g (H_0 + L \sin\varphi)}{dP/dt} \right)}{\ln(R_0/b)} dL.$$

By performing integration over L, we obtain

$$U \frac{\ln(R_0/b)}{2\pi\lambda} = \left(t_{bf} - t_a - \frac{U_{tot}}{\alpha S \eta} - \frac{\rho_L g H_0}{dP/dt} \right) L_0 + \frac{\rho_L g \left(L_0^2 \sin\varphi \right)}{2 dP/dt},$$

or, what is the same thing:

$$U \frac{\ln(R_0/b)}{2\pi\lambda} = \left(t_{bf} - t_a - \frac{U_{tot}}{\alpha S \eta} - \frac{\rho_L g \bar{H}}{dP/dt} \right) L_0, \tag{9}$$

where H is the average excess of the liquid level in the condenser, given by the ratio:

$$\bar{H} = H_0 + L_0 \sin \varphi.$$

Let N be the number of pipes of the evaporation system connected to the condenser. Multiplying both parts of Eq. (9) by N, we get

$$U_{tot} \frac{\ln(R_0/b)}{2\pi\lambda} = \left(t_{bf} - t_a - \frac{U_{tot}}{\alpha S \eta} - \frac{\rho_L g \bar{H}}{dP/dt} \right) L_{tot}. \tag{10}$$

It is taken into account here that the following relations are fulfilled:

$$U_{tot} = UN,$$
$$L_{tot} = LN,$$

where L_{tot} is the total length of the evaporator pipes, m.

From Eq. (10), we find the value of the total thermal power

$$U_{tot} = \left(t_{bf} - t_a - \frac{\rho_L g \bar{H}}{dP/dt} \right) \left[\frac{1}{\alpha S \eta} + \frac{\ln(R_0/b)}{2\pi\lambda L_{tot}} \right]^{-1}. \tag{11}$$

Now we will consider the integral solution. The amount of total heat from the phase transition released by the soil during freezing of the cylinder with radius R_0 and length L_{tot} is written as

$$Q_{bf} = \sigma\pi R_0^2 L_{tot}, \quad \sigma = \sigma_0 \gamma (w - w_0),$$

where Q_{bf} is the heat of the phase transition, J; σ_0 is the specific heat of ice melting, J/kg; γ is the density of the rock matrix, kg/m^3; w is the total moisture content of the rock, unit fraction (u.f.); w_0 is the rock moisture due to unfrozen water, u.f.

The amount of heat Q_t that leaves the system due to temperature change:

$$Q_t = \int_b^{R_0} \left(c_1 (t_0 - t_{bf}) + c_2 (t_{bf} - t(r)) \right) L_{tot} 2\pi r dr, \tag{12}$$

where $t(r)$ is the temperature at a point with radius r, °C; c_1 is the volumetric heat capacity of thawed soil, J/(m^3·°C); c_2 is the volumetric heat capacity of frozen soil, J/(m^3·°C); t_0 is the initial temperature of the soil, °C.

The solution of Eq. (1) can be written as

$$t(r) = \frac{t_{bf} - t_{ev}}{\ln(R_0/b)} \ln\left(\frac{r}{b}\right) + t_{ev}. \tag{13}$$

Substituting (13) into (12), we get

$$Q_t = \left(c_1 (t_0 - t_{bf}) + c_2 (t_{bf} - t_{ev}) \right) L_{tot} \pi R_0^2 - c_2 L_{tot} \frac{t_{bf} - t_{ev}}{\ln(R_0/b)} \int_b^{R_0} \ln\left(\frac{r}{b}\right) 2\pi r dr.$$

Thus, the energy balance equation takes the form

$$\int_0^{\tau} U_{tot}(\tau') d\tau' = Q_t + Q_f, \tag{14}$$

where τ is the time, days.

Differentiating both parts of Eq. (14) by τ, we obtain

$$U_{tot} = \frac{dR_0}{d\tau} \left(\frac{\partial (Q_t + Q_{bf})}{\partial R_0} \right). \tag{15}$$

Note that the solution of the integral given below can be written as

$$\int_b^{R_0} \ln\left(\frac{r}{b}\right) 2\pi r dr = 2\pi b^2 \int_1^{\frac{R_0}{b}} x \ln(x) dx = 2\pi b^2 \left(F\left(\frac{R_0}{b}\right) - F(1) \right), \tag{16}$$

where $F(x)$ is a function that is given by the expression

$$F(x) = \frac{x^2}{2} \ln(x) - \frac{x^2}{4}. \tag{17}$$

Thus, the right side of Eq. (15), taking into account expressions (16) and (17), will be written as

$$\frac{\partial\left(Q_t+Q_{bf}\right)}{\partial R_0}=2\pi R_0 L_{tot}\left(\sigma+c_1\left(t_0-t_{bf}\right)\right)+$$

$$+c_2 L_{tot}\frac{t_{bf}-t_{ev}}{\left(\ln\left(R_0/b\right)\right)^2}\frac{2\pi b^2\left(F\left(R_0/b\right)-F(1)\right)}{R_0}. \quad (18)$$

Note that the following substitution can be made in expression (18):

$$2\pi b^2\left(F\left(\frac{R_0}{b}\right)-F(1)\right)=\pi R_0^2\ln\left(\frac{R_0}{b}\right)-\pi\frac{R_0^2}{2}+\frac{\pi b^2}{2}.$$

Thus, we get

$$\frac{\partial\left(Q_t+Q_{bf}\right)}{\partial R_0}=2\pi R_0 L_{tot}\times$$

$$\times\left[\sigma+c_1\left(t_0-t_{bf}\right)+c_2\left(t_{bf}-t_{ev}\right)\times\right.$$

$$\left.\times\frac{\left(\ln\left(R_0/b\right)-0.5+0.5b^2/R_0^2\right)}{2\left(\ln\left(R_0/b\right)\right)^2}\right]. \quad (19)$$

To assess the correctness of the obtained solution, we check the convergence of the right side of Eq. (19) under the condition $R_0\to b$, i.e., under the condition that there is no frozen soil near the evaporation pipes.

Assuming $R_0 = b + x$, we get

$$\lim_{x\to 0}\frac{\ln\left(R_0/b\right)-0.5+0.5b^2/R_0^2}{2\left(\ln\left(R_0/b\right)\right)^2}=$$

$$=\lim_{x\to 0}\frac{x-0.5x^2-0.5+0.5\left(1-2x+6x^2\right)}{2x^2}=\frac{2.5}{2}.$$

Thus, Eq. (19) converges to a finite positive value, which indicates the correctness of the solution.

To simplify the expressions obtained, we introduce the function

$$\varphi(x)=\frac{\ln(x)-0.5+0.5/x^2}{\left(\ln(x)\right)^2}. \quad (20)$$

Then, we obtain the following differential equation from (11), (15), (19), and (20):

$$\left(t_{bf}-t_a-\frac{\rho_L g\bar{H}}{dP/dt}\right)\left[\frac{1}{\alpha S\eta}+\frac{\ln\left(R_0/b\right)}{2\pi\lambda L_{tot}}\right]^{-1}=\frac{dR_0}{d\tau}2\pi R_0 L_{tot}\times$$

$$\times\left(\sigma+c_1\left(t_0-t_{bf}\right)+c_2\left(t_{bf}-t_{ev}\right)\varphi\left(\frac{R_0}{b}\right)\right),$$

or, which is the same:

$$d\tau\left(t_{bf}-t_a-\frac{\rho_L g\bar{H}}{dP/dt}\right)=dR_0\frac{2\pi R_0}{\lambda}\left(\frac{\lambda L_{tot}}{\alpha S\eta}+\frac{\ln\left(R_0/b\right)}{2\pi}\right)\times$$

$$\times\left(\sigma'+c_2\left(t_{bf}-t_{ev}\right)\varphi\left(\frac{R_0}{b}\right)\right), \quad (21)$$

where the value σ' is given by the ratio

$$\sigma'=\sigma+c_1\left(t_0-t_{bf}\right).$$

Let us consider the case, when the following condition is met:

$$c_2\left(t_{bf}-t_{ev}\right)\varphi\left(\frac{R_0}{b}\right)\ll\sigma',\quad \alpha=\text{const}.$$

Then, Eq. (21) can be interpreted explicitly:

$$\tau\left(t_{bf}-\overline{t_a}(\tau)-\frac{\rho_L g\bar{H}}{dP/dt}\right)=\frac{\pi\left(R_0^2-b^2\right)}{\lambda}A\sigma'+$$

$$+\frac{b^2}{\lambda}\left(\left(\frac{R_0^2}{2b^2}\ln\left(\frac{R_0}{b}\right)-\frac{R_0^2}{4b^2}\right)+0.25\right)\sigma'. \quad (22)$$

Here, the values of $\overline{t_a}$ and A are given by the following relations:

$$\overline{t_a}(\tau)=\frac{1}{\tau}\int_0^\tau t_a(\tau')d\tau',$$

$$A=\frac{\lambda L_{tot}}{\alpha S\eta}.$$

Thus, an analytical solution has been obtained that can be applied to evaluate the efficiency of the functioning of the "HET" type soil temperature stabilization system. It is also worth noting that the proposed analytical solution is based on the integral formulation of the problem given in [*Naterer, 2003*].

ASSESSMENT OF CORRESPONDENCE BETWEEN ANALYTICAL AND NUMERICAL SOILUTIONS

Comparison of the numerical solution with experimental data [*Ishkov et al., 2018*] showed a high degree of agreement between theory and practice, which confirms the viability of the developed model.

To assess the degree of agreement between the data obtained via solving the numerical and analytical models, let us consider the freezing of soils by the "HET" system for various climatic zones. For this, meteorological parameters were taken from archival data for the Arctic cities of Varandey, Salekhard, and Igarka. The mean monthly temperatures and wind speeds for these cities are given in Table 1.

It should be noted that the efficiency of the HET system was evaluated for a number of variable parameters that can be combined into two large groups: climatic (air temperature, wind speed) and technical

Table 1. **Mean monthly air temperature (t_a) and wind speed (v) in settlements**

Month	Varandey		Salekhard		Igarka	
	t_a, °C	v, m/s	t_a, °C	v, m/s	t_a, °C	v, m/s
January	–14.7	6.9	–22.9	2.2	–26.1	3.1
February	–18.9	6.3	–19.2	2.2	–18.2	2.6
March	–13.0	6.2	–12.7	2.8	–14.6	2.9
April	–7.1	5.7	–5.5	3.2	–1.9	3.2
May	–1.5	5.6	1.1	3.4	3.9	3.4
June	6.1	5.6	11.6	3.5	11.4	3.1
July	10.8	5.8	16.0	2.9	15.8	2.8
August	9.2	6.1	11.5	2.9	11.2	2.8
September	6.5	5.7	6.3	2.9	7.1	3.1
October	1.5	6.9	–2.5	2.7	–4.2	3.3
November	–7.8	5.7	–13.4	2.5	–19.2	2.8
December	–21.3	2.8	–17.6	2.1	–24.4	3.2

(total length of evaporator pipes). All other parameters of the HET system and environmental conditions for each of the solutions have constant values.

There can be much more parameters, but those that have the greatest impact on the efficiency of the system were chosen. Thus, we get:

– Total length of evaporator pipes L_{tot} = = 300–3000 m;

– Air temperature depending on the region t_a = –34.1...+16.0°C;

– Wind speed depending on the region v = = 2.1–6.9 m/s.

The dynamics of the last two parameters (t_a, v) are different for each region for which the calculations of the system functioning are carried out. The calculation of the operating time of the HET system for each region, except for Varandey, has been carried out from the beginning of October (the first month with a subzero temperature). For Varandey, the calculation starts from November.

The radius of the frozen soil around the evaporator pipe acts as an output parameter by which the numerical and analytical solutions are compared and a conclusion is made about the efficiency or inefficiency of the HET system.

It should be noted that the discreteness of the obtained values of the soil freezing radius is 1 day for the numerical solution and 1 month for the analytical solution.

In all calculations, it is assumed that the soil has the following thermal characteristics: γ = 1600 kg/m^3, w_0 = 0, w = 0.2, λ = 2.0 W/(m·°C), c_1 = = 1.60·10^6 J/(kg·°C), c_2 = 1.47·10^6 J/(kg·°C).

The heat transfer coefficient between the condenser and the atmosphere is given by the following expression [*Royzen, Dulkin, 1977*]:

$$\alpha(t) = 0.105\,\frac{\lambda_a(t)}{s}\left(\frac{d}{s}\right)^{-0.54}\left(\frac{h}{s}\right)^{-0.14}\left(\frac{vs}{\nu_a(t)}\right)^{0.72},$$

where s is the distance between the capacitor fins, m; d is the diameter of condenser pipes, m; h is the length of the rib condenser, m; $\lambda_a(t)$ is the thermal conductivity of air, W/(m·°C); $\nu_a(t)$ is the kinematic viscosity of air, Pa·s; and v is the wind speed, m/s.

For the installation presented in this study, the characteristics of the condenser have the following values: d = 32 mm, s = 7 mm, h = 34 mm, S = 100 m^2, H_{con} = 5 m. The viscosity and thermal conductivity of air depend on the temperature of the atmosphere and are set according to the reference data [*Babichev et al., 1991*].

Thus, solving Eq. (22) for the meteorological parameters of Salekhard, we obtain the dependence of the radius of the soil freezing on time within the framework of the analytical and numerical solutions (Table 2).

Their graphic representations are given in Fig. 3.

The solutions for the meteorological parameters of Varandey are presented in Table 3 and Fig. 4; analogous solutions for Igarka, in Table 4 and Fig. 5.

After analyzing the results obtained from the numerical and analytical models, we can conclude that, on average, for all lengths of the evaporator, the degree of correspondence between the values of the soil freezing radius obtained by the two methods is 97.3% (Table 5).

It should be noted that for the evaporator length of 300 m, the difference in soil freezing radii between the numerical and analytical solutions is 10.5%, while for all other lengths it does not exceed 8%. Of course, there is a certain error, but it is worth recalling that the use of an analytical model is much simpler than the use of a numerical one, so the resulting accuracy is acceptable. Thus, the use of an analytical model to evaluate the efficiency of the HET-type soil temperature stabilization system for various design solutions and climatic conditions is quite justified and is suitable for quick assessment.

Table 2. **Dependence of soil freezing radius R_0 on time τ for Salekhard**

τ, days	R_0, m (analytical solution)				R_0, m (numerical solution)			
	Evaporator length							
	300 m	600 m	1000 m	3000 m	300 m	600 m	1000 m	3000 m
1	0.016	0.016	0.016	0.016	0.059	0.048	0.040	0.027
20	0.200	0.179	0.158	0.109	0.205	0.174	0.148	0.095
40	0.361	0.328	0.295	0.209	0.378	0.327	0.282	0.185
60	0.526	0.481	0.436	0.314	0.544	0.474	0.412	0.274
80	0.673	0.619	0.563	0.409	0.690	0.603	0.524	0.350
100	0.803	0.741	0.676	0.495	0.820	0.719	0.627	0.420
120	0.928	0.858	0.784	0.577	0.945	0.831	0.726	0.488
140	1.021	0.945	0.865	0.639	1.039	0.914	0.800	0.540
160	1.090	1.010	0.926	0.685	1.110	0.978	0.858	0.580
180	1.141	1.058	0.970	0.719	1.165	1.029	0.903	0.613
200	1.165	1.081	0.991	0.735	1.190	1.052	0.924	0.628

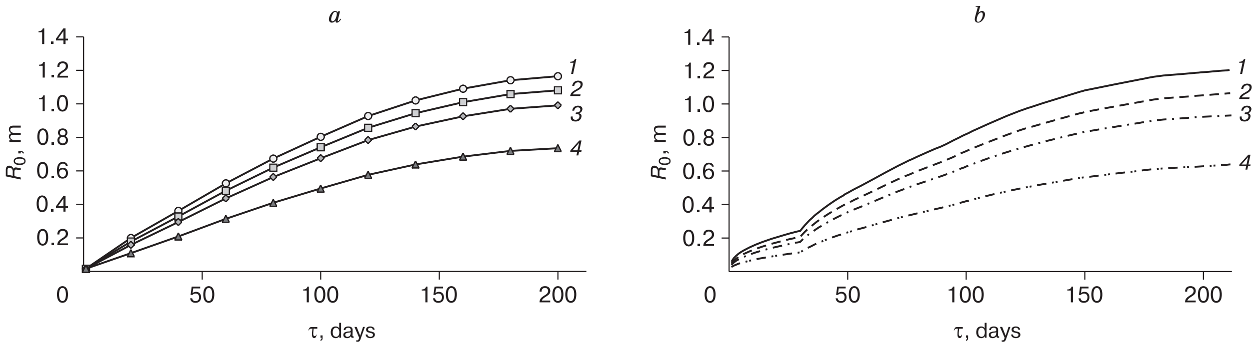

Fig. 3. Dependence of the freezing radius R_0 on time τ for Salekhard.

a – analytical solution; *b* – numerical solution. Evaporator length: *1* – 300 m; *2* – 600 m; *3* – 1000 m; *4* – 3000 m.

Table 3. **Dependence of soil freezing radius R_0 on time τ for Varandey**

τ, days	R_0, m (analytical solution)				R_0, m (numerical solution)			
	Evaporator length							
	300 m	600 m	1000 m	3000 m	300 m	600 m	1000 m	3000 m
1	0.016	0.016	0.016	0.016	0.057	0.048	0.041	0.028
20	0.172	0.160	0.148	0.110	0.183	0.163	0.144	0.099
40	0.304	0.286	0.267	0.206	0.328	0.296	0.265	0.185
60	0.437	0.414	0.388	0.305	0.465	0.423	0.380	0.270
80	0.564	0.536	0.505	0.400	0.603	0.554	0.503	0.365
100	0.676	0.644	0.607	0.485	0.723	0.667	0.609	0.447
120	0.784	0.748	0.706	0.567	0.837	0.774	0.709	0.524
140	0.896	0.856	0.810	0.653	0.955	0.886	0.812	0.603
160	0.983	0.940	0.890	0.720	1.045	0.970	0.890	0.662
180	1.047	1.002	0.949	0.769	1.112	1.033	0.948	0.706
200	1.084	1.037	0.983	0.798	1.151	1.068	0.981	0.731

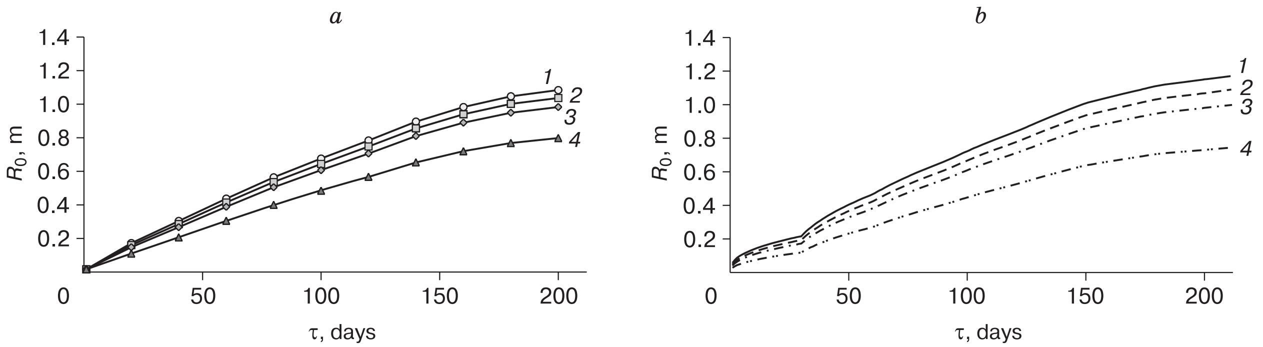

Fig. 4. Dependence of the freezing radius R_0 on time τ for the city of Varandey.

a – analytical solution; *b* – numerical solution. Evaporator length: *1* – 300 m; *2* – 600 m; *3* – 1000 m; *4* – 3000 m.

Table 4. **Dependence of soil freezing radius R_0 on time τ for Igarka**

τ, days	R_0, m (analytical solution)				R_0, m (numerical solution)			
	Evaporator length							
	300 m	600 m	1000 m	3000 m	300 m	600 m	1000 m	3000 m
1	0.016	0.016	0.016	0.016	0.078	0.062	0.052	0.034
20	0.253	0.230	0.207	0.146	0.264	0.228	0.197	0.130
40	0.440	0.405	0.369	0.268	0.463	0.406	0.355	0.238
60	0.632	0.585	0.536	0.396	0.654	0.578	0.508	0.344
80	0.800	0.744	0.684	0.510	0.829	0.739	0.654	0.449
100	0.939	0.875	0.806	0.604	0.972	0.870	0.773	0.535
120	1.061	0.990	0.913	0.688	1.098	0.985	0.877	0.610
140	1.141	1.066	0.985	0.744	1.178	1.057	0.941	0.655
160	1.205	1.126	1.041	0.788	1.243	1.116	0.993	0.691
180	1.258	1.176	1.088	0.824	1.298	1.166	1.038	0.723
200	1.269	1.187	1.098	0.832	1.308	1.175	1.046	0.729

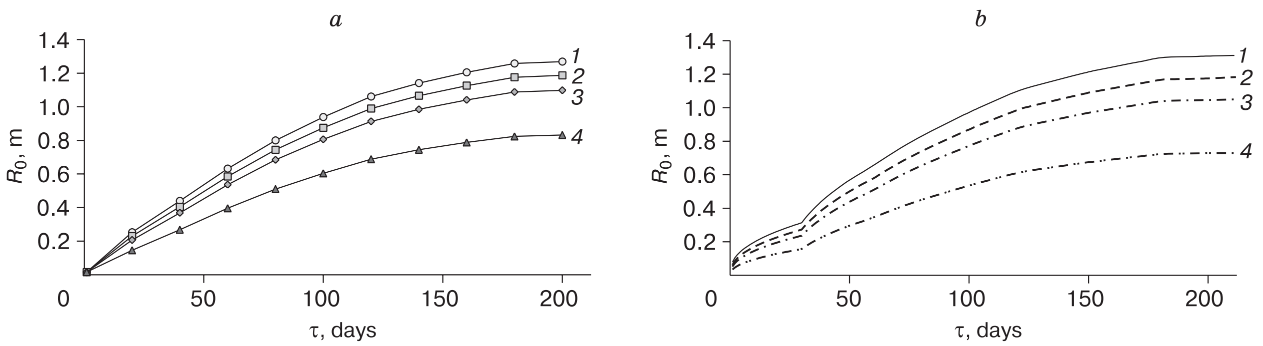

Fig. 5. Dependence of the freezing radius R_0 on time τ for the city of Igarka.

a – analytical solution; *b* – numerical solution. Evaporator length: *1* – 300 m; *2* – 600 m; *3* – 1000 m; *4* – 3000 m.

Table 5. **Degree of correspondence between analytical and numerical solutions for the radius of frozen soil (unit fraction)**

City	Evaporator length			
	300 m	600 m	1000 m	3000 m
Salekhard	0.912	0.963	1.010	1.114
Varandey	0.878	0.913	0.949	1.049
Igarka	0.896	0.939	0.979	1.075
Average	0.895	0.938	0.979	1.079

It follows from the presented data that if the distance between the pipes of the HET system is 1 m, then the entire soil will freeze in 100 days, i.e., in half of the winter season, since the freezing radius will usually be more than 0.5 m during this time However, for the Varandey case at L_{tot} = 3000 m, the radius of the frozen ground is 0.485 m, which is close to 0.5 m.

In addition, according to the calculated values of the soil freezing radius, it is possible to make a quick assessment of the volume of soil frozen under the object. To do this, we calculate the volume of the resulting soil cylinder with an evaporator laying step of 1 m. We get

$$V_{tot} = \pi R_0^2 LN = \pi R_0^2 \frac{L_{tot}}{N} N = \pi R_0^2 L_{tot},$$

where V_{tot} is the total volume of frozen soil, m^3; L is the length of one pipe of the evaporative system, m; N is the number of pipes in the evaporator system.

Thus, with a finning area of the condenser part of 100 m^2, in 100 days of operation of the HET-type soil temperature stabilization system with a total length of the evaporative part of 3000 m, it is almost always possible to freeze soil with a volume of 2356 m^3.

CONCLUSIONS

1. An analytical model of the functioning of the system for thermal stabilization of soils of the HET type was developed on the basis of the integral method.

2. A comparison of the results of numerical and analytical solutions for the soil freezing radii for different Arctic cities (Varandey, Salekhard, Igarka) demonstrated a high degree of correlation between the results obtained.

3. Based on the comparison of numerical and analytical solutions, it was concluded that the developed analytical model can be used for quick evaluation of the functioning of the HET-type soil temperature stabilization system for various design solutions and climatic characteristics.

4. A method for estimating the volume of frozen soil based on the data obtained within the framework of the analytical model solution is shown.

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