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METHODS OF CRYOSPHERIC RESEARCH

A PROBABILISTIC MODEL FOR PREDICTING SANDSTONE STRENGTH USING ELECTROMAGNETIC INDUCTION SOUNDING IN THE SOUTHERN YAKUTIAN PERMAFROST REGION: A CASE STUDY IN NERYUNGRI

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This paper presents a retrospective analysis of the geometric electromagnetic induction (EMI) sounding data. The data were acquired in the 1990s in the city of Neryungri to determine probabilistic relationships between unconfined compressive strength of saturated sandstone samples and the attenuation coefficient of the harmonic field induced by a high-frequency vertical magnetic dipole at 1.125 MHz in frozen sandstone massif. The results indicate that the consistent increase in the attenuation coefficient with decreasing strength of sandstone massif is correctly described by a logistic function equation. The inverse regression relationship is adequately described by a power function equation which can be used as a probabilistic model for predicting mean values of unconfined compressive strength of saturated sandstone massif (but not only sandstone rock samples) from the attenuation coefficient. The relative error of model predictions at the 70–80% confidence level is ±(27.7–32.0)%, which is close to the limit of allowable error (±20%) for laboratory measurements of mean strength of rock samples. This provides favorable conditions for applying the geometric EMI method in rock strength mapping for geotechnical engineering in Neryungri, as well as in areas of similar geology in southern Yakutia with sporadic permafrost.

Keywords*: strength, sandstone massif, geometric electromagnetic induction sounding, field of high-frequency vertical magnetic dipole, amplitude decrease coefficient, statistics, histograms and variograms, probabilistic model, prediction error.*

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INTRODUCTION

V.S. Yakupov with coauthors [*1998*] attached great importance to probabilistic models seeing their great possibilities in solving petrophysical problems by geoelectric methods in the cryolithozone of Russia. This class of physical-geological models, to the development of which G.S. Vakhromeev made a great pioneer contribution, is very informative and is urgently needed for the most complete and reliable solution of forecasting problems by geophysical methods. These models integrate knowledge about the natural probabilistic relationships between the geological and geophysical characteristics of the Earth rather than represent particular isolated facts. V.P. Melnikov drew attention to the need of application of geophysical methods to solve the problems of predicting the state of permafrost [*1977, p. 60*]: *"Information is needed about how the parameters of the electromagnetic field characterize not the individual elements of the part of the section we are interested in, but the site and the type of landscape as a whole. The evaluation criteria can be the types of vertical electrical sounding curves*".

The solution to one of the geomechanical problems of frozen soil – the problem of probabilistic prediction of the average strength of a frozen high-temperature sandstone massif according to the geometric electromagnetic induction (EMI) sounding method – is discussed in this paper. This problem is important for design and survey geological works. Two study cases are considered: sandstone massif at the base of engineering structures in Neryungri and sandstone massif composing the sides of quarries of mineral deposits in the South Yakutian Basin.

The forecast problem is solved by the classical way of comparing and linking the values of two geological-geophysical characteristics to one another. From the side of geology, the laboratory characteristic R_c is considered – the time limit of strength for uniaxial compression of sandstone samples in a water-saturated state (strength characteristic R_c). From the geophysics' side, the characteristic of the attenuation of the harmonic high-frequency field of a vertical magnetic dipole (HFVMD) in a frozen high-temperature sandstone massif is considered. The coefficient of reduction of the amplitude of the vertical component of the field depending on the spacing of the dipole installation of the EMI method (*k*) is taken as a quantitative estimate of the measure of the HFVMD field integral attenuation within the layer of annual temperature fluctuations. A.T. Akimov wrote about the paramount importance of the layer of annual temperature fluctuations for the construction and operation of engineering structures [*1971, p. 10*], figuratively and accurately calling it a "factory" of cryogenic processes.

The results of a retrospective analysis of the EMI data obtained in Neryungri in the 1990s and presented in this article fully justify the choice of the coefficient *k*. For the first time, the statistical relationships of the coefficient *k* with the R_c are studied. On this basis, a previously unknown probabilistic model designed to predict the average strength of a sandstone massif in a water-saturated state was built. This model complements the results of petrophysical studies in the sporadic permafrost zone of southern Yakutia using well logging [*Grib, Samokhin, 1999*] and the georadar method [*Neradovsky, Fedorova, 2015; Neradovsky, Syasko, 2015*].

STUDY AREA

The only comprehensive work on permafrost in southern Yakutia is the study by a team of researchers from Moscow State University headed by V.A. Kudryavtsev [*Kudryavtsev, 1975*]. It is rich in factual materials and their versatile systematization. This work provides information about all aspects of the natural environment of southern Yakutia before its industrial development. According to this study, the research area, including the city of Neryungri, is located in the folding zone along the marginal suture of the Cis-Stanovoy Trough in the southern margin of the Aldan Plateau within the Chulman Depression composed of coal-bearing sedimentary rocks of the Jurassic and Lower Cretaceous. In the southern part of the Chulman Plateau (in the middle course of the Chulman River), deposits of the Lower Cretaceous Kholodnik Formation (sandstones with layers of conglomerates, siltstones, and coal) lie on the sediments of the Gorkita Formation (sandstones with interlayers of conglomerates, siltstones, coal), with the former formation serving as the core of the Neryungri syncline with fault dislocations. This tectonic factor put a specific imprint on the permafrost in Southern Yakutia [*Buldovich et al., 1976*]. In general, the research area is characterized by complicated engineering and geocryological conditions, including heterogeneity and dynamism of permafrost soil conditions. This natural feature manifests itself in a significant depth of seasonal thaw–freeze processes (about 3–6 m) with the possibility of multiple transition of frozen rocks into a thawed state and vice versa.

The city of Neryungri – the administrative center of South Yakutia – is located 800 km southwest of Yakutsk. The city occupies the top and slopes of the watershed of the Chulman, Upper Neryungri, Maly Berkakit, and Amnunnakta rivers. The absolute heights of the watershed vary from 773 to 868 m a.s.l. The watershed surfaces have mild permafrost conditions with the development of sporadic permafrost of small thickness (20–50 m) with the mean annual temperature in the lower part of the layer of annual temperature fluctuations close to 0°C. The natural temperature regime of permafrost is largely controlled by the convective heat transfer processes. The movement of air and water from the surface and from the depths of the Earth along the bedrock fracture zones leads to a decrease in the permafrost thickness, as well as to an increase in the thickness of the layer of annual temperature fluctuations. According to various estimates , the lower boundary of this layer lies at a depth of 15–30 m.

Unstable and spatially variable natural temperature regime of the bedrock in Neryungri has been disturbed during the construction and operation of engineering facilities. Since 1975, complete degradation of permafrost in some of the city sites has taken place. According to the generalized thermometry data from survey boreholes, the temperature at the permafrost degradation sites at a depth of 10 m varies from 0 to 6.5°C. In areas of preserved permafrost with a thickness of 13–18 m, the mean annual temperature at a depth of 10 m is –(0.4 ± 0.1)°C.

The analysis of data obtained by the South Yakutian Engineering and Construction Survey Center (Yuzhyakuttisiz) allows us to make a number of generalizations in relation to the geological structure of the territory of Neryungri.

First, the thickness of colluvium covering hard bedrock is about 2–3 m. Second, the bedrock of sedimentary genesis lies with a small dip angle. Third, up to a depth of 6–7 m, the bedrock is strongly loosened by physical weathering; below, to the maximum depth of drilling (10–20 m), its strength state is preserved. Fourth, the bedrock is composed mainly of sandstone, which is found in almost every borehole with a probability of 97%. The detecting probability of other rocks – interlayers or layers of siltstone, carbonaceous mudstone, and coal layers – is 18%, 8%, and 29%, respectively. Fifth, blocks of medium strength are most often found in the sandstone massif. The occurrence probability of sandstone blocks of low, reduced, or high strength is 26%, 41%, 50%.

These data are in agreement with the data on the coal-bearing rocks of the Aldan–Chulman Coal Basin. For example, according to V.M. Zhelinsky [*1980*], sandstone (fine-medium-coarse-grained) prevails in all the suites of Southern Yakutia. Its share varies from 52.6% in the Durai suite to 86.5% in the Yukhta suite. In the Neryungri Formation, spatially close to the territory of Neryungri, the proportion of fine- to medium-grained sandstone is 78.7%.

METHOD OF GEOMETRIC ELECTROMAGNETIC INDUCTION SOUNDING

The study of the HFVMD attenuation in a frozen high-temperature sandstone massif was carried out within the layer of annual temperature fluctuations by the EMI method with the medium-frequency electromagnetic sounding (MEMS) equipment manufactured by the NPO Sibtsvetmetavtomatika (Krasnoyarsk). This measurement technique made it possible to study all the components of the polarization ellipse of the HFVMD at four fixed frequencies (0.281; 0.562; 1.225; 2.250 MHz) and to determine the effective values of the electrical resistance ρ_{ef} and permittivity ε_{ef} from them [*System..., 1991*]. The modern analogue of the MEMS equipment is the digital KAV–EMM equipment. Until 2016, it was manufactured by R D Science LLC in Krasnoyarsk with the participation of employees of the Siberian Federal University. In the 1990s, the MEMS equipment was considered unique in terms of technical and economic parameters both in centers of engineering and construction surveys and in geological exploration enterprises. The antennas of the MEMS equipment have a round shape with an external diameter of 32 cm. The current in the transmitting antenna is regulated by the power supply block from 0 to 100 μA. The dynamic range of the microvoltmeter is 70 dB (from 0.5 to 1000 μV).

In Russia, the electromagnetic induction sounding method was classified as a geometric variant of the methods of AC geoelectrics. Abroad, it is referred to as the electromagnetic induction (EMI) method. Most often, EM-31 and EM-38 ground conductivity meters are used. A general overview of the EMI equipment is given in in the work of J. Boaga [*2017*].

The EMI method in Russia has been applied since the 1969s; the history of its development in Russia is briefly described in [*Igolkin et al., 2016*]. In addition, interesting and instructive information about the development of the inductive methods in the Urals is available on the website of the Bulashevich Institute of Geophysics of the Ural Branch of the Russian Academy of Sciences. In this institute, the technology of the EMI method was born through the efforts of the head of the Laboratory of Electrical Induction Survey G.V. Astrakhantsev and his colleague V.S. Titlinov. Almost unchanged, it has been used for many years with the equipment of DEMP-SF, AFS-78 and MFS-8, MFS-10 in the search for copper ore deposits and for solving other geological problems.

The EMI method is described in detail in [*Veshev et al., 1978; Zhuravleva et al., 1994; Titlinov, Zhuravleva, 1995; Zaderigolova, 1998*]; special guidelines have been published [*Frantov, 1984*].

The methodology for studying the process of attenuation of the HFVMD in the layer of annual temperature fluctuations has been successfully tested for many years in the Republic of Sakha (Yakutia) and partially in the Trans-Baikal Territory, Amur Region [*Neradovsky, 2018*] and consists of the following. At the sounding site, at a height of about one meter, a transmitting antenna with a constant frequency of HFVMD is fixed parallel to the earth surface. The receiving antenna is removed from the transmitting antenna at distances from 3–5 m to 50–60 m by consecutive steps (2–5 m). With the horizontal position of the receiving antenna, the amplitude of the vertical component (H_z) of the HFVMD is measured with a microvoltmeter of the MEMS equipment; in the vertical position of the receiving antenna, the values of the horizontal component (H_r) are measured. The measurement result obtained at a height equal to the height of the transmitting antenna is attributed to the standing point of the transmitting antenna [*Frantov, 1984*]. The combination of H_z, H_r measurements (or other combination of the components of the HFVMD polarization ellipse) depends on the distance between the antennas and is commonly called the EMI signal.

The experience of using the EMI method in different engineering and geological conditions of the cryolithozone of Yakutia suggests that in almost all cases, a regular nonlinear decrease in the values of the H_z and H_r amplitudes is observed on the EMI signal graphs and is described by the power function equation with a high multiple determination coefficient (R^2). The value of the exponent of the power function is assigned to the coefficient k. This is the main difference between the EMI method and its usual application in Russia and abroad for the sole purpose of studying the effective electrical resistance (ρ_{ef}) in the kilohertz frequency range. Of course, it is necessary to study this static characteristic of the electric field, which is the most important in geoelectrics. However, the result of the study in relation to solving problems of geology will becomes fuller with the addition of new knowledge on the attenuation of the HFVMD in the layer of annual temperature fluctuations. Alas, this most important and, in addition, poorly studied side of the energy interaction of the HFVMD with an inhomogeneous anisotropic geological medium is not interesting to geophysicists. Since the end of the 1970s, foreign geophysicists have been expanding the boundaries of the scientific and practical application of the EMI method in order to study ρ_{ef} without going beyond the limits of the kilohertz frequency range. This is seen from the review paper [*Doolittle, Brevik, 2014*]. Only in some early works (e.g., [*Sartorelli, French, 1982*] and from the report of the Alaska Department of Transportation and Public Facilities [*McNeill, 1980*], in which only fleeting attention is paid to the dynamic side of the EMI signal that is considered just as an inductive response from the layers of the lower half-space[1]. However, this is done one-sidedly in the context of studying ρ_{ef} without studying the attenuation of the vertical magnetic dipole (VMD) field. The reason for the reluctance of Russian and foreign colleagues to study the attenuation of this field at low and, especially, high frequencies is not exactly known. Perhaps, this is hindered by the stereotype of thinking in terms of the mathematical and physical incorrectness of solving this problem in the conditions of the screening influence of the primary VMD field on the weak inductive response of the geological medium in the secondary field[2]. Surely, in the near zone of the primary field source, its influence on the secondary field is so strong that it prevents obtaining correct and reliable knowledge about the petrophysics of the lower half-space. The influence persists in the transitional zone of the source, where it is weakened to such an extent that it no longer prevents reliable study of the induction effect in the total field in dependence of the geological setting [*Neradovsky, 2018*].

In Neryungri, the work by the EMI method was carried out in the1990s during the experimental and methodological experiments in order to study the engineering and geological capabilities of this new and promising method. According to the recommendations of geophysicists of Yakutzoloto (Yakutian Gold) Company, who had extensive practical experience with the EMI method, the study of the sandstone massif was carried out at a frequency of 1.125 MHz and a spacing of 5–50 m with the measurements of H_z and H_r values for the sole purpose of determining the values of ρ_{ef} from their ratio. With a retrospective analysis of the data of the EMI method, the goal has changed. Only the vertical component of the HFVMD was studied in order to determine the values of the k coefficient. The reason for this is explained by the higher geological informativeness of H_z in comparison with H_r. According to A.A. Petrovsky [*1971*], H_z carries about 70% of information about anomalous objects that are located in the lower half-space. Moreover, practical experience shows that H_z is less dependent on the noise influence (topography, engineering structures, etc.).

As for the H_r and H_z measurements, they were made in the spring, i.e., in the period, when the seasonally thawing colluvium layer was completely frozen. Owing to this, the investigation depth of the sandstone massif by the EMI method increased. The distorting influence of the seasonally thawed layer on the HFVMD attenuation in this massif disappeared.

The effective penetration depth of the HFVMD into this layer (h_{ef}) was estimated for the entire territory of Neryungri by the values of the reduced distance parameter (P) equal to the product of the modulus of the complex wave number (ζ) by the spacing values. The values of ζ were calculated according to the formulas of electrodynamics of continuous media. They are not difficult to find in any of the numerous textbooks on electrical exploration and therefore they are not given in this article.

The generalized estimates of the average effective values of electrical conductivity, as well as the real part of the complex permittivity (ε_{ef}) necessary for the calculation were determined in different ways. The conductivity was estimated by $1/\rho_{ef}$. The average value of ρ_{ef} was calculated from the ensemble of H_z/H_r measurements at the EMI points with the conversion of the H_z/H_r ratio to the corresponding parameter N. Implicitly, this parameter takes into account the frequency influence of ε_{ef} on ρ_{ef}. The values of N for H_z/H_r values in the tabulated form are presented in the work by V.I. Igolkin with coauthors [*2016, p. 260–266*]. The formula for calculating ρ_{ef} is given in the same paper in the following form:

$$\rho_{ef} = Nr^2 f, \qquad (1)$$

where N is the parameter of the dielectric constant, rel. units; r is the spacing, m; and f is the frequency, MHz.

For example, for the H_z/H_r value equal to 1.5 rel. units measured at a distance of 15 m and at a frequency of 1.125 MHz, we find N = 1.2 rel. units in the specified table [*Igolkin et al., 2016, p. 263*]. Substituting it into formula (1) together with the values of the spacing and frequency, we get a ρ_{ef} equal to 303.8 Ω·m.

Unfortunately, the value of ε_{ef} in the presented procedure for determining ρ_{ef} remains unknown. If, during the experimental and methodological works, measurements were made of the large and small axes of the polarization ellipse of HFVMD and of the angle of inclination to the horizon of the large axis, then the values of ε_{ef} could have been determined using special pallets and nomograms [*Lebedev et al., 1991*] together with the values of ρ_{ef}. However, the lack of proper experience at the initial stage of mastering the MEMS equipment and the EMI method prevented the implementation of such a possibility. The missing values of ε_{ef} were found in the course of georadar surveys in Neryungri with an OKO-2 georadar and the Triton antenna unit operating at a central frequency of 50 MHz [*Neradovsky, 2022*].

The values of ε_{ef} were determined in a standard way by the angle of inclination of the branches of the

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[1] This is what geophysicists call the geological medium in theoretical studies.

[2] The author of this article witnessed a similar situation when, in the mid-1980s, scientific and production tests of the first domestic samples of ground-penetrating radars were initiated by the Committee on State Construction of the Russian Federation. At that time, the absolute majority of geophysicists who worked not only in engineering and geological surveys but also in geological exploration categorically denied the real facts of the depth resolution up to 20–30 m of soundings in frozen soils at a frequency of tens to hundreds of megahertz.

hodographs of diffracted waves. The results of the determination of ε_{ef} by this method are confirmed by an independent control check. It was made in Moscow by R.R. Denisov with the help of the well-known computer program "Georadar-Expert" developed by him. According to georadar data, the ε_{ef} values for the layers were distributed in depth as follows. In an undifferentiated layer of road pavement soils, colluvial deposits, and a strongly weathered upper part of sandstone, the values of ε_{ef} to a depth of 5–7 m vary in a narrow range from 4.12 to 4.88 rel. units. In the lower, relatively intact part of the sandstone massif, the values of ε_{ef} at a depth of 7–28 m are equal to 3.5–4.0 rel. units. The decrease is caused by a change in the genesis and composition of geological formations with a natural decrease in the amount of clay material in deeper layers. In general, the average value of ε_{ef} according to georadar data is 3.93 rel. units. This value obtained at a frequency of 50 MHz is further used to calculate the module ζ at a frequency of 1.125 MHz. The difference in frequency is very large, but it can be ignored without harm to the merits of the case, guided by the unspoken decision of the international community of geophysicists – possibility of using theoretical concepts of the so-called "georadar platform" in georadiolocation, where the values of ε_{ef} almost do not change in a wide frequency range from 1 MHz to 1 GHz [*Vladov, Sudakova, 2017*].

As for the electrical conductance characteristic $1/\rho_{ef}$, the generalized estimate of its average value at a frequency of 1.125 MHz and in the spacing range of 5–50 m is 1/2800 Ω·m (0.000357 S). The actual values of ρ_{ef} are determined from palettes and nomograms [*Lebedev et al., 1991*] based on the totality of H_z/H_r values determined at all points of the EMI, in which the attenuation of the HFVMD in Neryungri was studied. The calculation based on the average values of $1/\rho_{ef}$ = 0.000375 S and ε_{ef} = 3.93 rel. units showed that for this city the generalized value of the module ζ is 0.062 m^{-1}. The values of P equal to 0.311 and 3.11 rel. units correspond to the spacing values of 5 and 50 m. Do they satisfy the criteria according to which the boundaries of the intermediate zone of the source of the primary HFVMD favorable for work by the EMI method are established?

Theoretically, the boundaries of the intermediate zone are set according to the criterion $P << 1$ (the neighborhood of the near zone) and $P >> 1$ (the neighborhood of the far wave zone). G.V. Molochnov and M.V. Radionov [*1983*] consider it correct to use the EMI method to designate the boundaries of the intermediate zone by P values in the range of 1–10. V.I. Igolkin et al. [*2016*] link the boundaries of the intermediate zone with the accuracy of measuring the components of the HFVMD. These researchers establish the boundaries of the transitional zone according to the criterion $0.01 \leqslant P \leqslant 100$ with an accuracy of up to 1%. In the field, it is impossible to ensure such accuracy, which means that, strictly speaking, it is impossible to apply the proposed criterion. Thus, what criterion should be chosen? Let us make a choice based on a general theoretical criterion. Although it is fuzzy, it is applicable in many cases. In accordance with this criterion, the above defined values of P equal to 0.311 and 3.11 rel. units, correspond to the boundaries of the intermediate zone, in which the problem of probabilistic prediction of the strength of the sandstone massif according to the EMI method is theoretically solved correctly.

To determine the desired generalized estimates of the average values of h_{ef}, we turn to the little-known work of V.S. Titlinov and R.B. Zhuravleva [*1995, p. 13*], borrowing from it a graph of the theoretical dependence of the h_{ef}/h_s ratio on the parameter P. The theoretical value of the maximum thickness of the skin layer (h_s) (skin depth or penetration of the HFVMD at a frequency of 1.125 MHz) is determined by a simple formula given in the same work with reference to the textbook on electrical survey by M.S. Zhdanov:

$$h_s = \sqrt{2}/\zeta, \qquad (2)$$

where h_s is the thickness of the skin layer, m; ζ is the modulus of the wave number, m^{-1}.

Setting ζ equal to 0.062 m^{-1} in formula (2), we get h_s = 22.8 m. With a value of P = 0.311 rel. units, according to the above graph, we find h_{ef}/h_s = 0.26. From this ratio, in turn, we calculate the value of h_{ef} = 22.8·0.26 = 5.9 m. Similarly, for P = 3.11 rel. units and the corresponding h_{ef}/h_s = 0.78, we find h_{ef} = 17.8 m.

Thus, for Neryungri, the accepted frequency (1.125 MHz) and the spacing range (5–50 m) provide correct determination of the attenuation of HFVMD within a relatively intact lower part of the sandstone massif at a depth of 5.9–17.8 m. This depth corresponds to the depth of the geological study of the strength of the massif in the core samples.

SANDSTONE STRENGTH

In laboratory environments, the quantitative assessment of the strength of sandstone samples taken from air-dry or frozen core samples was carried out according to R_c values. This strength characteristic is widely and universally used in Russian soil mechanics. In foreign geotechnics, it is applied together with other geological and mining characteristics in order to build probabilistic models. For example, a neural model of multiple regression in order to predict the velocity of borehole drilling with a diamond bit in the fields of Turkey [*Basarir et al., 2014*].

From the thousands of boreholes drilled by YuzhYakuttisiz during the construction development of Neryungri, R_c values were collected from only 218 boreholes, but this amount of factual material is

Table 1. **Statistics of classification of laboratory values of the time limit of strength for uniaxial compression of sandstone samples in a water-saturated state**

Parameter	R_c, MPa		
	groups 2–4	group 5	group 6
Arithmetic mean	10.5	34.8	65.5
Median mean	11.5	38.0	58.9
Modal mean	18.0	16.2	54.2
Weighted mean	10.5	30.8	65.9
Standard deviation	12.1	11.1	16.4
Coefficient of variation, %	31.4	32.7	25.1
Minimum value	2.5	15.0	50.2
Maximum value	14.8	49.0	114.6
Sample size	46	104	68

Note: Groups 2–4 to sandstone of low, reduced, and moderately low strength (R_c = 15 MPa); group 5, to moderately strong sandstone (R_c = 15–50 MPa); and group 6, to strong sandstone (R_c = 50–120 MPa).

sufficient for a reliable assessment of probability distributions of R_c values using the “Stage” program [*Kulaichev, 2006*]. As for the error in determining the average R_c values in the YuzhYakuttisiz soil laboratory, it is not known exactly. However, according to GOST (State Standard) 21153.2-84 [*1984*], with mass laboratory determinations in a series of six samples taken from each core, the error cannot be higher than ±20%.

With the arithmetic mean and median R_c values of 39.1 and 40.7 MPa, the particular values varied from 2.5 to 114.6 MPa with the variation coefficient of 59.6%. In 70% of cases, R_c values were distributed near the averages in the range of 15.8–62.4 MPa. According to [*GOST 25100-2020, 2020*], in the ordered sequence of R_c increase in Neryungri, the sandstone massif is represented by five strength groups. The first and second groups (sandstone of low and reduced strength with R_c equal to 1–5 MPa) are rarely present ((probability 2.7%). The probability of the third group (low-strength sandstone with R_c = 5–15 MPa) is higher and equals 19.3%. The fourth group of moderately strong sandstone with R_c = 15–50 MPa has the maximum probability of 47.7%. The fifth group of strong sandstone (R_c = 50–120 MPa) has a lower probability of occurrence (30.7%).

The group statistics of R_c values are presented in Table 1. With an abnormal distribution of R_c values, the correct theoretical indicator of the average distri-

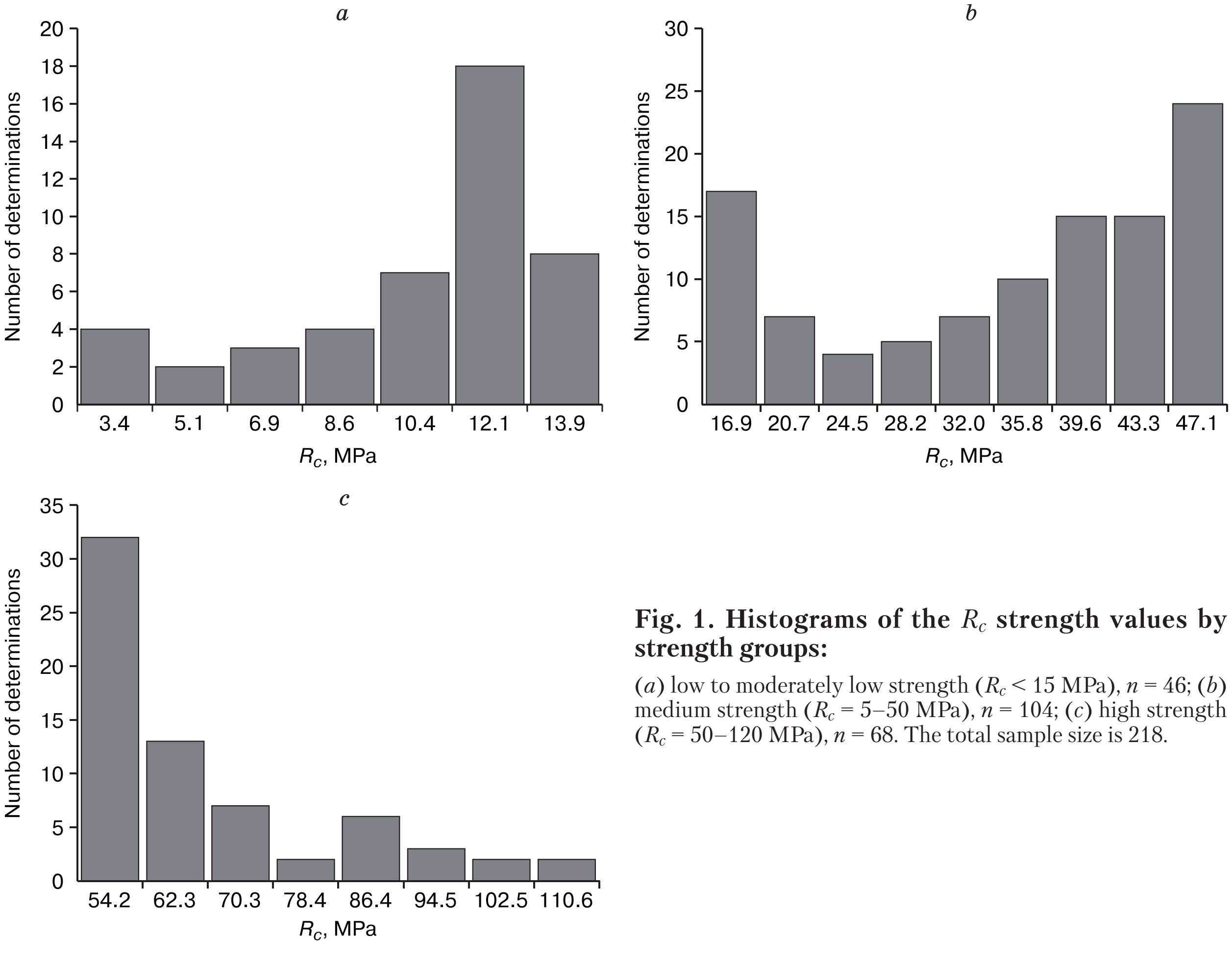

Fig. 1. Histograms of the R_c strength values by strength groups:

(*a*) low to moderately low strength (R_c < 15 MPa), n = 46; (*b*) medium strength (R_c = 5–50 MPa), n = 104; (*c*) high strength (R_c = 50–120 MPa), n = 68. The total sample size is 218.

bution position is the median average value. The proximity of the arithmetic mean and weighted average values to the median average value serves as a true indication that, in Neryungri, the background distribution of the strength values of the sandstone massif obeys the normal law. However, with a small number of determinations in group samples (Table 1), the background of the R_c distribution is complicated by the imposition of anomalous components on it. As a result, an inhomogeneous probabilistic structure is formed. It manifests itself as isolated maxima at the ends of histograms with abnormally low (below 10.4 MPa (Fig. 1*a*)) or high (above 70.3 MPa (Fig. 1*c*)) R_c values with a small frequency of their occurrence.

The areas of abnormal R_c values are considered as rare events indicating the appearance of two opposite groups of sandstone massif at the base of engineering structures in Neryungri: low and high strength. Most often, at the base of engineering structures there are sections of sandstone massif of medium strength and partly of low strength. These events are detected by the maxima of the general histogram of the R_c characteristic (Fig. 2). The first maximum with a mode of 10–20 MPa identifies a joint group of a sandstone massif of low, reduced, and moderately low strength. In terms of construction, they are considered as an undifferentiated category of semi-rocky soils. The second and third maximums with modes 40–50 and 80–90 MPa indicate the existence of two strength groups in the sandstone massif forming the dominant category of rocky soils. These are the groups of moderately strong and strong sandstone.

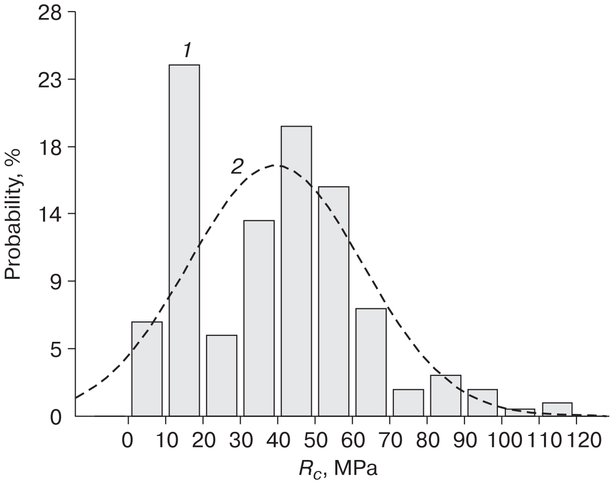

Fig. 2. Summary of the (*1*) actual histogram and (*2*) theoretical variogram of the law of the normal probability distribution of the R_c strength characteristic values.

Sample size, 218 determinations.

VERTICAL MAGNETIC DIPOLE FIELD

Recall that the values of the coefficient *k* are taken as a quantitative assessment of the integral measure of the attenuation of HFVMD in the sandstone massif within the layer of annual temperature fluctuations. The error in determining this coefficient is estimated from the results of control measurements of H_z. With an average relative error of measuring H_z values equal to 12.7%, a similar error in determining the values of the *k* coefficient was 7.6%.

In Neryungri, the values of the *k* coefficient vary from 1.08 to 4.41 m^{-1}. The width of the integral variability in the coefficient of variation is rather large and amounts to 42.3%. The average values of the *k* coefficient are close and equal to 2.31; 1.88; 1.63 m^{-1}. In 70% of cases, the values of the *k* coefficient vary near the average in the range of 1.33–3.29 m^{-1}.

Histograms constructed with reference to the boundaries of the rock – semi-rock soil classes show that the values of the *k* coefficient inherit from the strength of the sandstone massif an inhomogeneous probabilistic structure, which is not described by either the law of normal distribution or the laws of abnormal distribution (Fig. 3). The statistics of the values of the coefficient *k* obtained for the groups of different strength of the sandstone massif are presented in Table 2. It follows from this table that with a decrease in the strength of the sandstone massif, the average values of the *k* coefficient increase. However, this variability is different, both in terms of the averages and the coefficients of variation. With a general decrease in the average values of the R_c strength characteristic by 6.3 times, the average values of the *k* coefficient increase by 2.6 times. With a more even integral variability (by the coefficient of variation) of the values of the R_c strength characteristic with an average median value of 31.4%, the coefficient *k* is significantly lower and equals to 14.6%.

Thus, the response of the HFVMD attenuation to the variability in the strength of the sandstone massif in Neryungri is weakened by the action of other permafrost and soil factors, such as temperature, mineral composition, grain-size distribution, type and composition of cement, presence of impurities, and other petrographic characteristics.

The result of the attenuation is visible on the histogram of the values of the coefficient *k* constructed for the entire city of Neryungri (Fig. 4). In contrast to the same histogram of the R_c strength characteristic (Fig. 2) in the histogram of the *k* coefficient, there is no clear separation of the modes of the high and medium strength sandstone groups forming a mixed group with probable mode values in the range of 1.36–1.91 m^{-1}. The same group with a mode of 3.58–3.85 m^{-1} is observed on the histogram on the right, combining sandstone groups of low, reduced, and moderately low strength.

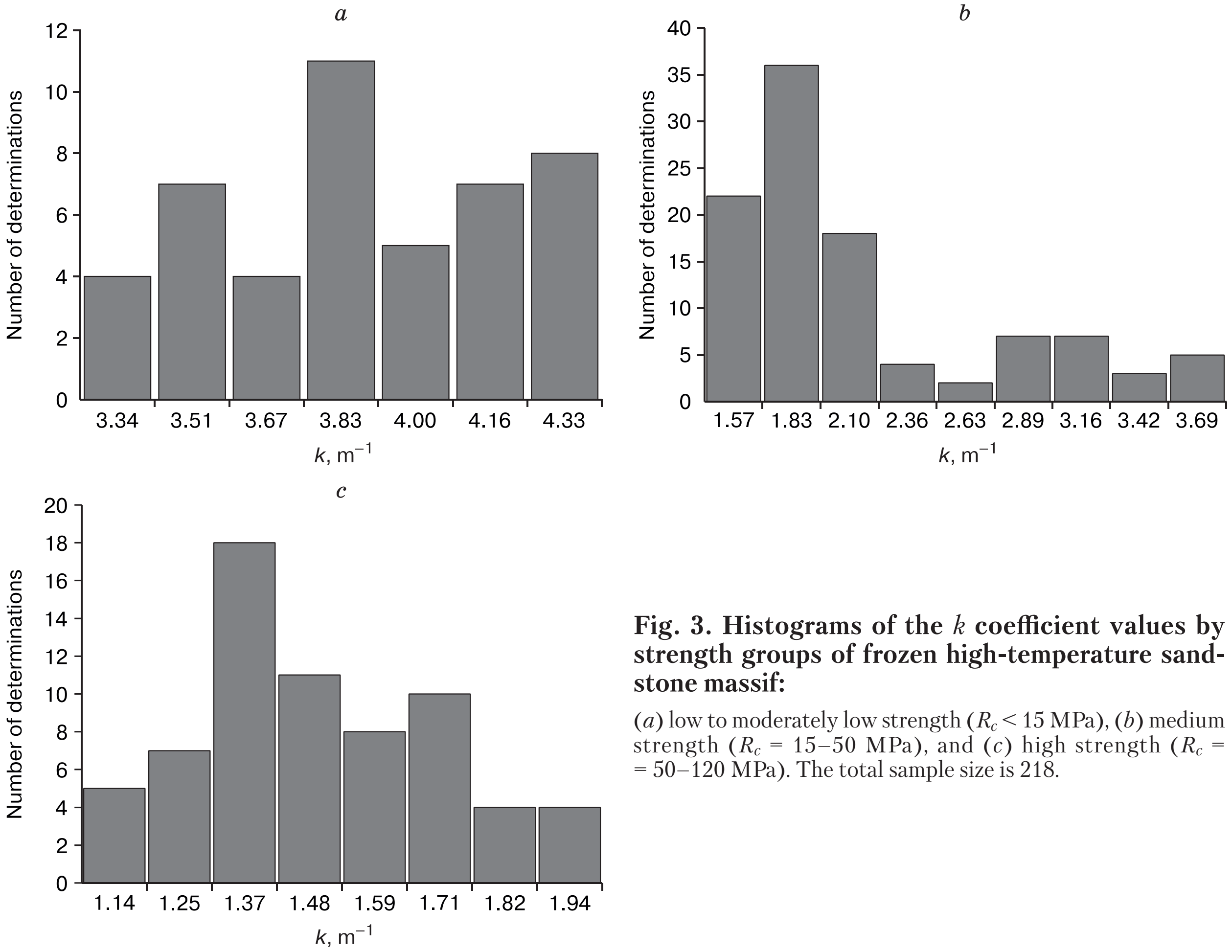

Fig. 3. Histograms of the *k* coefficient values by strength groups of frozen high-temperature sandstone massif:

(*a*) low to moderately low strength ($R_c < 15$ MPa), (*b*) medium strength (R_c = 15–50 MPa), and (*c*) high strength (R_c = = 50–120 MPa). The total sample size is 218.

Table 2. **Statistics of the HFVMD attenuation in a frozen sandstone massif according to the coefficient *k* values**

Parameter	*k*, m^{-1}		
	groups 2–4	group 5	group 6
Arithmetic mean	3.86	2.14	1.49
Median mean	3.84	1.91	1.44
Modal mean	4.26	3.12	1.35
Weighted mean	3.88	2.21	1.49
Standard deviation	0.33	0.62	0.22
Coefficient of variation, %	8.4	29.0	14.6
Minimum value	3.26	1.44	1.08
Maximum value	4.41	3.82	1.99
Sample size	46	104	68

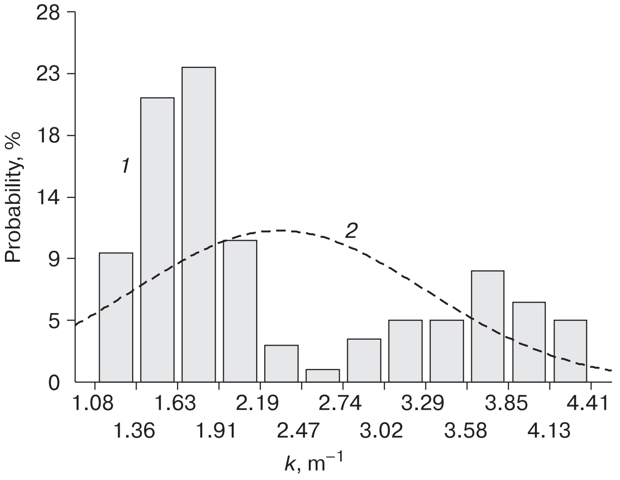

Fig. 4. Summary of the (*1*) actual histogram and (*2*) theoretical variogram of the law of the normal probability distribution of the coefficient *k* values for a frozen high-temperature sandstone massif.

Sample size, 218 determinations.

The features of the probabilistic structure of the distribution of the values of the coefficient k show the weak and strong side of the EMI method in solving the problem of predicting the average strength of sandstone. The weak side is the difficulty of reliable recognition, even by the average values of the k coefficient, of a group of strong sandstones and a group of medium-strength sandstones forming one category of rocky soils in terms of construction [*GOST 25100-2020, 2020*]. The strength of the EMI method is a reliable distinction of the category of rocky soils from the category of semi-rocky soils (groups of sandstones of moderately low, reduced, and low strength). It is clear that the strength of the EMI method is of particular importance for design and survey work, especially for solving one of the main tasks of engineering-geological zoning of the built-up territories of South Yakutia and Neryungri according to the strength of the foundations of engineering structures.

PROBABILISTIC MODEL

For a long time, the scale factor has been considered a fundamental obstacle to the application of probabilistic models linking geological and geophysical characteristics. The accumulated experience of field experiments shows that the incompatibility of point geological characteristics and volumetric geophysical characteristics is valid only in the space of disparate single determinations of these characteristics. When comparing a large number of values of geological and geophysical characteristics ordered by area and time, the dominant role of the scale factor loses its significance and does not interfere with detecting and studying statistical patterns [*Paskhaver, 1974*].

In the probabilistic model under consideration, in scientific terms, of course, the petrophysical side is interesting. It describes the correlation between the coefficient k and the strength characteristic R_c. In practical terms, another side of the model is important, which describes the physically unrealistic regression dependence of R_c on k. It is this side of the model that provides a valuable opportunity to move from laboratory determination of the time limit of strength for uniaxial compression of water-saturated sandstone samples to a quantitative assessment of the same strength for a sandstone massif, that is, to solve the desired problem of predicting the strength of a sandstone massif according to the EMI method.

According to the rules and assumptions used in mathematical statistics, random variations of an independent variable associated with errors in laboratory determination should be insignificant in a probabilistic model in comparison with the general range of variability [*Draper, Smith, 2007, p. 41*]. This is a very strict theoretical condition, but it is practically met for the strength characteristic R_c, which is considered from the correlation side of the probabilistic model as an independent and non-random variable. In fact, with the relative variability of R_c values equal to 191.5% (2.5–114.6 MPa), random variation of permissible laboratory errors of strength determination (no higher than ±20% according to [*GOST 21135.2-84, 1984*]) is only 10.4%.

The graph of the correlation dependence of k on R_c is shown in Fig. 5. It follows from this dependence that the increase in the strength of the sandstone massif is accompanied by a regular nonlinear decrease in the average measure of attenuation of the HFVMD. The graph shows three sections with boundaries of sandstone strength groups [*GOST 25100-2020, 2020*]. The first section with a weak attenuation change and a correlation coefficient of –0.70 corresponds to a group of sandstone of moderately low, reduced, and low strength ($R_c < 15$ MPa). The second section with maximum attenuation variability and correlation coefficient –0.74 corresponds to a group of moderately strong sandstone with R_c from 15 to 40 MPa. In the third section within the group of strong sandstone with $R_c > 40$ MPa, attenuation slows down again, but not as much as in the first section. At the same time, the correlation does not decrease and constitutes –0.79.

The described correlation is close to the cause-and-effect relationships that arise when the strength of the sandstone massif changes and the response to this variability of the measure of the HFVMD attenuation. It is difficult to say which mathematical functions correctly describe these relations, but with a high degree of probability, one of them is the logistic function. This function is known to correctly describe the behavior of natural-technical systems at their extreme transitions, i.e., from the initial state to the final state. In relation to the probabilistic model for Neryungri, the limiting transition is the transition from the low-strength sandstone to the strong sandstone in the rock–semi-rock basement of engineering structures.

The graph of the logistic function, constructed by the least squares method, is shown against the background of the scattering field of factual data on the paired values of R_c and k (Fig. 5). The general record of the equation of the logistic function for all cases has the following form:

$$x = \left[\left(a_0 + a_1\right) \big/ \left(1 + a_2 \exp\left(a_3 y\right)\right)\right] + \delta,$$

where y is an independent variable (characteristic R_c); x is a dependent variable (coefficient k); a_0, a_1, a_2, a_3 are parameters depending on the conditions of application of the EMI method; and δ is a random error.

In relation to the engineering and geological conditions of construction and operation of engineering structures in Neryungri, the values of a_0–a_3 are equal to 1.2050, 3.5930, 0.1413, and 0.0795, respec-

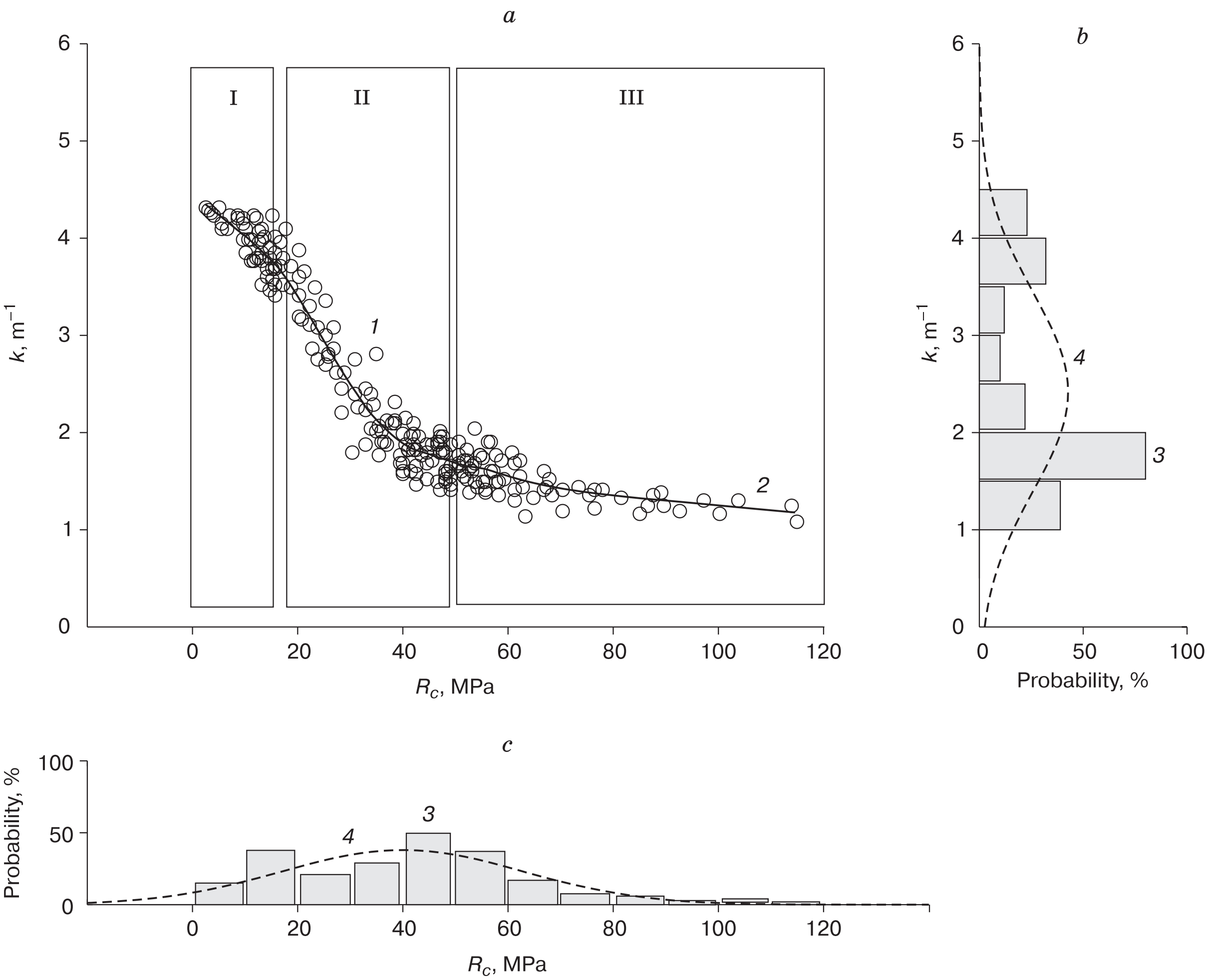

Fig. 5. A probabilistic model for strength groups of water-saturated sandstone massif in accordance with [*GOST 25100-2020, 2020*] with the (*a*) correlation graph and graphs of probability distributions of the (*b*) coefficient *k* values and (*c*) strength characteristic R_c:

(*1*) actual data, (*2*) graph of the logistic function equation, (*3*) actual histograms, and (*4*) theoretical variograms of the law of normal distribution. Strength groups of sandstone: (I) low to moderately low strength, (II) medium strength, and (III) high strength.

tively. They relate by probabilistic relations the strength of the sandstone massif and the average measure of the HFVMD attenuation with a multiple determination coefficient $R^2 = 0.959$. This means that the contribution of the strength factor to the variability of the attenuation of the HFVMD dominates among other factors and reaches almost 96%.

The graph of the logistic function equation is useful, because it makes it possible to solve a direct geophysical problem without calculations. Namely, at the design stage of the work, it is possible to estimate the measure of attenuation of the HFVMD by a priori data on the strength of the sandstone massif. In addition, knowing the values of the coefficient *k* according to the EMI method, it is easy to quickly solve two tasks in the field: (1) to obtain an initial assessment of the strength of the sandstone massif and (2) to construct a scheme of preliminary zoning of the studied territory by strength groups with division into categories of rocky and semi-rocky soils.

The inverse problem of geophysics – the prediction of the average strength of a water-saturated sandstone massif – is solved by a probabilistic model using a regression equation of a power function. It approximates the regular variability of the average values of the strength characteristic R_c depending on the values of the coefficient *k* in the field of random scattering of factual data on Neryungri better than other functions. The graph of the power function equation is demonstrated in Fig. 6. It follows from this graph that the permutation of the places of variables has changed the mathematical form, but not the essence of the probabilistic relations between R_c and *k*. Such a permutation should not cause confusion. It is inherent in regression analysis and occurs when the direction of transition from one space of probabilistic relations of variables to another space changes.

The regression equation of a power function has the following form with a multiple determination coefficient $R^2 = 0.793$:

$$R_c = \exp(4.836)\, k^{-1.707}. \tag{3}$$

Equation (3) is applicable for surveys by the EMI method in the spring, when the seasonally thawing colluvial layer remains in the completely frozen state. Another condition for the correct application of this equation is the range of variability of the ensemble of the coefficient k values (1.08–4.41 m^{-1}). Outside these boundaries, the parameters of equation (3) may be different.

FORECAST ERRORS

The errors of the probabilistic forecast model according to the EMI method of determining the average strength of a water-saturated sandstone massif present the difference between the laboratory values of the strength characteristic R_c and the values calculated by equation (3). A test check made using the "Stage" program [*Kulaichev, 2006*] according to several independent criteria indicates that the probabilistic distribution of absolute and relative errors does not obey the normal law. This is also confirmed by the graphs of error histograms (Fig. 7). In 122 cases, errors have a positive sign and in 96 cases, a negative sign. Such an imbalance in the spread of single values near the averages is a mark of a certain tendency towards underestimating the calculated R_c values in relation to laboratory values. The proportion of prediction errors of the probabilistic model, comparable in magnitude to the maximum permissible error of laboratory determination (±20%), is significant. In the actual material consisting of 218 determinations, errors of 20–30% occur in 61–78% of cases (Fig. 7*a*). A decrease in the sensitivity of the k coefficient in the region of high values (above 4 m^{-1}) confined to an undivided group of low-strength sandstone, sandstone

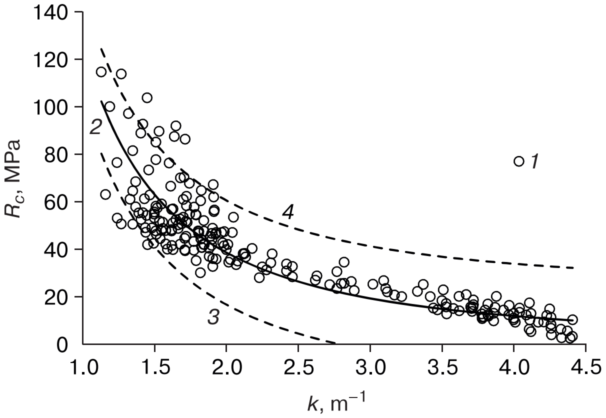

Fig. 6. Probabilistic model for predicting the average strength of a sandstone massif (R_c) in a water-saturated state from the values of coefficient k:

(*1*) actual material, (*2*) graph of the regression equation of the power function, and (*3*, *4*) lower and upper boundaries of the 95% confidence interval.

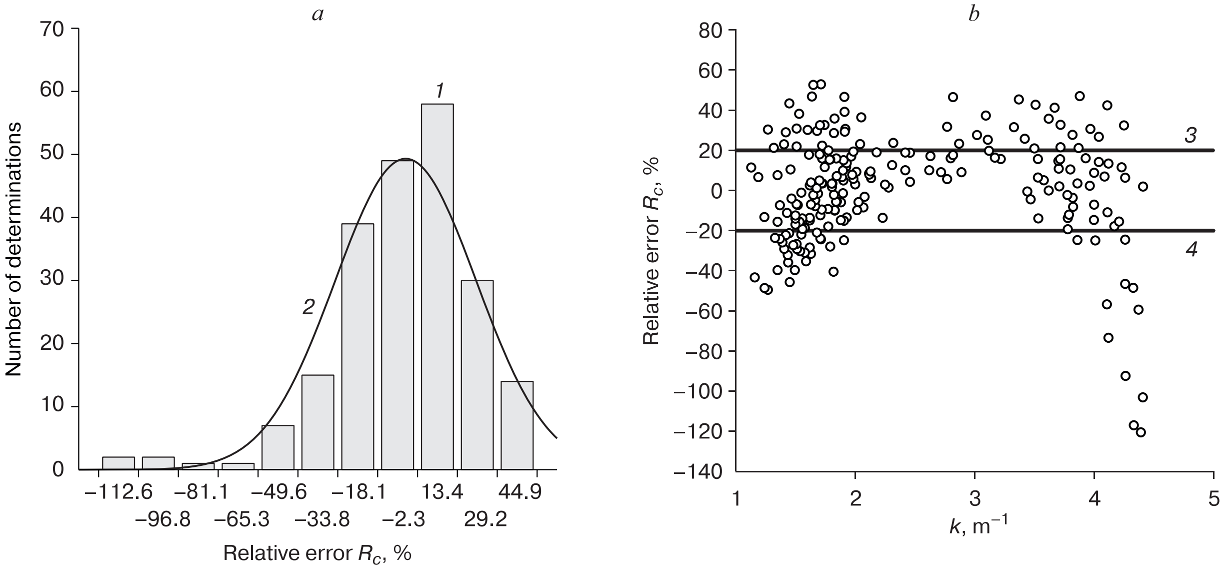

Fig. 7. Graph of the distribution of errors of the probabilistic model (*a*) and the scattering field of model errors depending on the coefficient k values (*b*).

(*1*) actual histogram of errors, (*2*) theoretical variogram of errors according to the law of normal probability distribution, and (*3*, *4*) limits of the maximum permissible error of laboratory determination of the R_c strength characteristic. Sample size, 218 determinations.

Table 3. **Statistics of errors of the probabilistic model of the average strength forecast of the sandstone massif in the water-saturated state according to the EMI method data**

Parameter	R_c error	
	MPa	%
Arithmetic mean	0.65	0.21
Standard error	0.76	1.88
Median mean	1.04	3.50
Modal mean	1.75	3.50
Standard deviation	11.24	27.78
Coefficient of variation, %	17.3	132.3
Minimum value	–34.7	–120.5
Maximum value	38.2	52.8
Sample size	218	218

of reduced and low strength with average calculated values of the R_c strength characteristic <15 MPa leads to an abnormal increase in errors up to –120%, i.e., to a systematic excess of the laboratory values of the R_c strength characteristic (Fig. 7*b*). The share of such errors is equal to 8.2–9.6%. A more favorable pattern of error distribution for the EMI method is observed at relatively low values of the coefficient *k* (<1.8 m^{-1}) for a group of strong sandstone with an estimated $R_c > 50$ MPa. In this case, the maximum error does not exceed ±55%, and the share of such errors in relation to errors of ±(20–30)% is ±34.5 and ±20.7%.

It turns out that in about 7–8 cases out of ten, the expected forecast error will fall within the range of ±(20–30)%. With the variation of single errors from 1.75 MPa (3.5%) to 11.24 MPa (27.8%), the average error rates are close to zero (Table 3). If we take the moderate level of confidence[3] in the results of geophysics equal to about 70%, which has been accepted for a long time in geology in the prospecting and exploration of mineral deposits, then the calculation errors of the average R_c values will be concentrated in the range ±11.22 MPa (±27.7%). At the confidence level of 80%, the errors become slightly larger and equal to ±14.07 MPa (±32.0%).

CONCLUSIONS

A retrospective analysis of the factual material obtained in the 1990s in the permafrost of southern Yakutia in Neryungri allowed us to build a new probabilistic model. This model makes it possible to solve an important problem of geomechanics of frozen soils in terms of predicting the average strength of a frozen high-temperature sandstone massif that has passed into a water-saturated state under the influence of anthropogenic, technogenic and climatic factors on the basis of data obtained by the EMI method. With a probability of 70–80%, the model allows us to estimate the strength of the sandstone massif in this state with a relative error of ±(27.7–32.0)%. This error is comparable with the permissible laboratory error of ±20%. Such a level of accuracy gives grounds to apply the EMI method for the purpose of quick (ahead of expensive and labor-intensive drilling and laboratory works) economical and detailed zoning of the territory of Neryungri according to the strength category of rocky–semi-rocky foundations of engineering structures.

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[3] This is a relatively low level of confidence, but sufficient to assess the reliability of the results obtained by geophysical methods taking into account their geological ambiguity and mathematical incorrectness of solving inverse problems.

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