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METHODS OF CRYOSPHERIC RESEARCH

COMPARATIVE ANALYSIS OF AREA DISTRIBUTIONS FOR THERMOKARST LAKES WITHIN DIFFERENT TYPES OF THE SURFACE OF THERMOKARST PLAINS WITH FLUVIAL EROSION

A.S. Victorov*, V.N. Karpalova, T.V. Orlov

Sergeev Institute of Environmental Geoscience, Russian Academy of Sciences, Ulansky per. 13, str. 2, Moscow, Russia 101000

**Corresponding author; e-mail: dist@geoenv.ru*

Thermokarst plains with fluvial erosion include two genetically different types of surface: slightly undulating watersheds with primary termokarst lakes and lowered surfaces of khasyreys (drained thermokarst lakes) with secondary lakes. The research deals with a comparative analysis of statistical distributions of the areas of thermokarst lakes and secondary lakes. Using statistical criteria and remote sensing data for eight key sites in different natural conditions, such as Yamal and Tazovsky peninsulas, the Kolyma Lowland, and the Penzhina River valley, we determined statistically significant differences in the area distributions of thermokarst lakes within different genetic types of the surface. Statistical analysis shows that the areas of thermokarst lakes correspond to an integral-exponential distribution. This allows us to conclude that a dynamic equilibrium is established within each type of the surface in the course of the initiation, growth, and drainage of thermokarst lakes. Though the parameters of thermokarst lakes differ significantly, we find a correlation between the distribution parameters of lake areas within the main surface of thermokarst plains with fluvial erosion and the surface of khasyreys with secondary lakes.

Keywords: thermokarst plains with fluvial erosion, mathematical morphology of landscape, khasyrey, alas, thermokarst lakes.

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INTRODUCTION

A significant number of studies are devoted to the development of thermokarst lakes on plains in the permafrost zone. Within the Permafrost Region Pond and Lake Database (PeRL) project, information is collected about the boundaries of thermokarst lakes obtained from high-resolution satellite images. Studies of thermokarst lakes based on the analysis of satellite images from different times are also being developed [*Olefeldt et al., 2016*].

Vast and numerous research dedicated to the morphological study and inventory of lakes in general is currently being conducted. Thus, in [*Verpoorter et al., 2014*] approximately 1.7 million lakes larger than 0.002 km2 were analyzed throughout the world, and the dependence of their quantity, surface area, and perimeter on latitude and elevation a.s.l. was described. In other works [*Cael, Seekell, 2016*] size distribution and boundary fractality (based on the relationship of perimeter to surface area) were studied for a large number of lakes throughout the world and compared to the distribution of lakes in Sweden. About 70 sites in Western Siberia were analyzed with respect to lake size distribution [*Polishchuk et al., 2018*]. Other authors performed analogous studies

also. In particular, an analysis of shoreline sinuosity and lake area was made; it was concluded that the sinuosity of a shoreline decreases as lake area increases [*Muratov et al., 2021*]. It should be noted that many authors [*Verpoorter et al., 2014; Cael, Seekell, 2016; Polishchuk et al., 2018*] use either cartographic inventory methods or automated analysis of satellite images and do not separate lakes based on genesis and type. Consequently, lakes of different types are mixed in such works, which is important for a complete inventory of the number and areas of lakes, but limits possibility for the analysis of processes typical for the studied objects in light of their polygenetic nature.

Similar study is being conducted by Chinese researchers for the Tibetan Plateau [*Wei et al., 2021*]. They analyze thermokarst lakes in mountain conditions. It should be emphasized that the authors do not make conclusions about the law of lake surface area distribution, possibly because of a large number of analyzed lakes existing under different conditions.

Local and regional research of the state and dynamics of thermokarst lakes is also being conducted. A study of lakes in the Kolyma Lowland demonstrated their increase by 4.5% during 1999–2018 [*Vereme-*

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eva et al., 2021]. For Yukon plains, a decrease in the area of lakes from the 1970s to the 1990s was identified along with an increase in their number [*Lantz, Turner, 2015*]. The number of thermokarst lakes in the studied territory increased by 10% over the span of 60 years, while the overall lake area and the number of large lakes decreased [*Jones et al., 2011*]. A decrease in the number of lakes in the Siberian part of the Arctic was reported in [*Smith et al., 2005*]. In the area of Eight Mile Lake (Canada), new primary thermokarst landforms appeared in about 12% of local landscapes [*Belshe et al., 2013*]. At the same time, upon analysis of remote sensing data for Yukon plains, the presence of multidirectional tendencies in changing lake sizes was identified: a 14% increase by 1994 and a 10% decrease during 1999–2002 compared to 1984 [*Chen et al., 2014*].

Changes in the morphological pattern of permafrost landscapes have been considered in relatively few works [*Kravtsova, Bystrova, 2009; Morgenstern еt al., 2011; Polishchuk V., Polishchuk Yu., 2013; Grosse еt al., 2016; Polishchuk et al., 2018*]. Furthermore, thermokarst plains and thermokarst plains with fluvial erosion have not been properly separated. Under thermokarst plains with fluvial erosion the present article implies vast surfaces with thermokarst depressions and a well-developed erosional network, which separates them from thermokarst plains with lakes, where the erosional network is significantly less developed. Fully or partly, drained thermokarst basins are known as khasyreys in Western Siberian and as alases in Yakutia.

Finally, a series of studies have attempted to create a mathematical model for the morphological pattern of thermokarst plains with fluvial erosion [*Victorov, 2005, 2006; Victorov et al., 2016*]. However, the proposed model has certain disadvantages, to which attention was brought during discussions at

conferences, including by V.E. Tumskoy. These disadvantages were associated, first of all, with the assumption that the progression is the same for the processes of initiation, growth, and drainage of thermokarst lakes forming on the main watershed surface and lakes forming on the surface of thermokarst depressions.

Thus, studies of thermokarst lakes aimed at identifying patterns in their development, including qualitative ones, usually insufficiently considered the peculiarities of lake development; lakes of thermokarst plains with and without a pronounced erosional network were often considered together, without making distinction between them. Meanwhile, the development of such lakes apparently occurs under different conditions and follows different patterns. Without knowledge about these patterns, it is difficult to continue studies of the development of thermokarst in permafrost landscapes and to conduct a retrospective analysis.

The goal of the present article is to present the results of a comparative analysis of statistical distributions of thermokarst lake areas within different genetic types of surfaces of thermokarst plains with and without fluvial erosion. An attempt is made to find general patterns typical for the corresponding type of surfaces at sites located in different physical-geographical, geological, and geocryological conditions.

METHOD FOR COMPARATIVE ANALYSIS OF AREA DISTRIBUTIONS OF THERMOKARST LAKES

Thermokarst plains with fluvial erosion represent undulating or slightly hilly surfaces with tundra or forest-tundra vegetation, with inclusions of thermokarst lakes and thermokarst depressions and the development of stream channels. Lakes and ther-

Fig. 1. Thermokarst plains with fluvial erosion on satellite images (Kolyma Lowland, WorldView2, 0.5 m/px, July 1, 2013).

 a – overall image; b – enlarged fragment; 1 – cluster of thermokarst depressions; 2 – main surface; white dotted line – boundary of a cluster of thermokarst depressions; black line – boundary of enlarged fragment.

mokarst depressions usually have rounded shapes and are irregularly distributed throughout the plain (Fig. 1*a*). A relatively elevated surface with the development of lakes is nominally called the main surface; this is the background element of topography and represents the first element of analysis in the present study (Fig. 1*b*).

Thermokarst depressions frequently merge together forming clusters; the lakes forming within them are largely secondary lakes that developed due to thermokarst process after the formation of the initial thermokarst depressions. We delineated these lakes based on clear, sharp outlines, often a rounded shape, and a discrepancy between the center of the lake and the center of the thermokarst depression. Contrarily, residual lakes in thermokarst depressions are characterized by blurred irregular outlines and vague boundaries. The aforementioned surface of thermokarst depressions, which are often joined (Fig. 1*b*), and the secondary lakes located in them are the second object of analysis (residual lakes are not considered).

Figure 2 shows satellite images of lakes on the main surface and secondary lakes on the surface of thermokarst depressions.

The study areas are located in different physicogeographical and geocryological conditions (Fig. 3) [*Geocryological Map..., 1991; State Geological Map…, 2000, 2015, 2016*].

The first group of sites is located on the territory of the Yamal and Tazovsky peninsulas with characteristic elevations of 20–70 m asl. The areas are predominantly composed of marine-alluvial and marine sediments of the Late Pleistocene–Holocene and Late Pleistocene ages. The upper part of the section is composed of fine- and medium-grained sand with in-

Fig. 2. Example of lakes seen on satellite image (Yamal Peninsula, image SPOT 7, 1.5 m/px, July 18, 2017).

1 – lakes on the main surface; 2 – secondary lakes on the surface of thermokarst depressions; 3 – white dotted line – boundary of thermokarst depression.

clusions of fine and medium gravel and with interlayers of loam, loamy sand, and clay; less often, loamy sand and loamy layers are present. This is the area of continuous permafrost with the mean annual temperature from -3 to -9° C is typical.

The second group of sites is found in the Kolyma Lowland with typical elevations of 20–70 m asl.

Fig. 3. Location of key sites.

Numbers – site numbers; black circles – site location; grey lines – boundaries of sites on the enlarged parts.

Loesslike lacustrine–alluvial sediments of the Late Pleistocene are overlain by lacustrine–bog sediments of the Early Holocene in thermokarst depressions. Lacustrine–alluvial sediments are silty and contain interlayers and lenses of peat. This is the area of continuous permafrost with the mean annual temperature from -7 to -11° C. The upper parts of the section are mainly composed of ice-rich sandy silt loams with massive cryostructure and large ice wedges.

The third group includes one site in the area of the Penzhina Bay (Parapolsky Dol) with typical elevations of 70–230 m asl. This territory is mainly composed of glaciofluvial sediments of the Middle Pleistocene: medium and fine gravels, sand, and boulders. Continuous permafrost has the mean annual temperature from -1 to -3 °C.

The methodology of research included the following stages:

– selection of key sites within thermokarst plains with fluvial erosion and obtaining satellite images;

– development of a mathematical model of the morphological pattern of thermokarst plains with fluvial erosion, taking into consideration the existence of two types of conditions for the development of thermokarst lakes: on the main surface of thermokarst plains and on the surface of thermokarst depressions;

– delineation of lakes and calculation of their area;

– selection of samples of thermokarst lakes formed on the main surface of thermokarst plains and in the already existing thermokarst depressions¹;

– comparison of empirical distributions of these types of lakes for each site using Smirnov's criterion;

– comparison of empirical distributions and theoretical distributions obtained using the model with calculation of distribution parameters;

– comparison of the values of lake area distribution parameters for thermokarst lakes of type 1 (on the main surface of thermokarst terrain) and type 2 (lakes in thermokarst depressions) and analysis of the relationships between them;

– comprehensive comparison and analysis of the obtained data.

Key sites were selected based on the homogeneity of physico-geographical and geocryological conditions, as well as morphological homogeneity. Satellite images taken in June–September 2013–2019 with a resolution of 0.5–1.5 m (WorldView-2, SPOT 6,7) were mainly used; for key site 22, Sentinel-2 images with a resolution of 10 m were used.

The formation of samples of the areas of thermokarst lakes of the first and second type for each site implied interpretation of satellite images, delineation of the lakes, and determination of their areas using modules of QGIS software. Delineation (vectorization) of the lakes (water area) was carried out manually by an operator. As indicated above, the lakes were separated into two types: type 1, isolated lakes on the main surface of thermokarst plain; type 2, lakes in the existing thermokarst depressions that often formed clusters.

The next stage implied the comparison of empirical distributions of lake areas for the two samples for each site. To estimate the significance of the differences, Smirnov's criterion was applied, as this criterion does not necessitate knowledge about the type of distribution.

Then, we compared the empirical and theoretical integral-exponential distributions using Pearson's goodness-of-fit criterion. The necessary condition for using this criterion – the number of values in each separated class of the sample should no less than 5, and the sample itself should include no less than 50 values – was met. Sample sizes varied from 87 to 350. Next, the values of the area distribution parameter for lakes of type 1 was compared with that for lakes of type 2. In particular, the correlation coefficients for parameter values were determined for all the sites.

During the final stage, analysis of the entire data set was performed.

RESULTS

The mathematical model of the morphological pattern of thermokarst plains with fluvial erosion [*Victorov, 2005, 2006*] was applied in this study with certain adaptation of basic assumptions for the considered conditions:

(1) The formation of initial thermokarst depressions (lake foci) on the main surface and on the surface of the already existing thermokarst depressions during non-intersecting periods of time (Δt) and in non-intersecting areas (Δs) are independent random events; the probability of the formation of depressions p_k (where k is the number of depressions) depends only on the length of the time period and on the size of the site2:

$$
p_1 = \lambda_i \Delta s \Delta t + o(\Delta s \Delta t), \quad i = 1, 2,
$$

$$
p_k = o(\Delta s \Delta t), \quad k = 2, 3...
$$

where λ_1 and λ_2 are the parameter values for the main surface and for the surface of thermokarst depressions, respectively.

(2) The formation of primary lakes does not occur on the surface of existing thermokarst lakes.

 $\frac{1}{1}$ For brevity, the former lakes are called type 1 lakes and the latter are called type 2 lakes.

² For small areas and short time periods, the probability of formation of one thermokarst depression is much greater than the probability of formation of several depressions.

(3) The radius of the formed thermokarst lake as a function of time is a random process; the change in the radius occurs independently of other lakes, and the rate of its growth is proportional to the density of heat losses through the lateral surface of the lake basin.

(4) During the growth process, a lake can transform into a thermokarst depression when drained through the erosional network; the probability of this process does not depend on other lakes; meanwhile, the growth of drained lakes stops.

(5) The formation of the sources of stream channels on non-intersecting areas is a random event with an average density of sources γ_1 and γ_2 for the main surface and for the surface of thermokarst depressions, respectively; the probability of the presence of such a source at the key site depends only on its size³.

In the basic version of the model, the density of lake generation and the distribution density of the sources of stream channels were assumed to be close both for the main surface and for the surface of thermokarst depressions, i.e., the entire key area was assumed to be relatively homogenous in terms of the indicated parameters [*Victorov, 2005; Victorov et al., 2016*]. It was shown that in a wide range of physicogeographical and geocryological conditions, given a significant time of development, the territory comes to a state of dynamic equilibrium [*Victorov, 2005, 2006*]. In this case, the initiation and growth of thermokarst lakes is compensated by their drainage and transformation into thermokarst depressions. The analysis of this basic version of the model indicates that the state of dynamic equilibrium is characterized by a specific type of distribution of the areas of thermokarst lakes, which was conventionally called the integral-exponential distribution [*Victorov et al., 2016, 2021*].

In the studied situation, upon analysis of the errors of the created model, it is easy to see that for each individual surface (the main surface and the surface of thermokarst depressions and their clusters), essentially, the conditions of the base model are met, i.e., each type of the surface types is relatively uniform in terms of the density of generation of lakes and sources of stream channels. In this case, using the base model, it can be concluded that, over a significant time, each type of the surface may come to a state of dynamic equilibrium. Consequently, the distribution of lake areas will be close to the integral-exponential distribution, with its own parameter values for each type of the surface:

$$
f_1(x,\infty) = -\frac{1}{x\mathrm{Ei}\left(-\gamma_1\varepsilon_1\right)}\mathrm{exp}\left(-\gamma_1x\right), \quad x \ge \varepsilon_1,
$$

$$
f_2(x,\infty) = -\frac{1}{x \mathrm{Ei}\left(-\gamma_2 \varepsilon_2\right)} \exp\left(-\gamma_2 x\right), \quad x \ge \varepsilon_2,
$$

where ε_1 , ε_2 are the initial lake areas for the main surface and for the surface of thermokarst depressions, respectively: γ_1, γ_2 are the average distribution density of the sources of stream channels for the main surface and for the surface of thermokarst depressions, respectively; and $Ei(-x)$ is the integral-exponential function. The values of corresponding parameters of lake area distributions for the two types of surface should differ from one another.

The analysis of this modified base model suggests that distributions of thermokarst lake areas on the main surface (f_1) and in thermokarst depressions (f_2) can correspond to a special type of distribution, which is nominally called integral-exponential, and this can be one of the elements of empirical verification.

Comparison of empirical distributions of lake areas in the selected samples with the use of Smirnov's criterion made it possible to estimate the statistical significance of the difference between distributions of thermokarst lakes of the first and second types regardless of the hypothesis about belonging to one or the other type of distribution. The results of this estimate are presented in Table 1 and Fig. 4. The analysis shows that at seven out of eight sites, the differences between distributions of the two types of thermokarst lakes are significant at the 0.99 level.

The next stage was a comparison of empirical and theoretical distributions. Following the analysis of the model of the morphological pattern development for the thermokarst plains with fluvial erosion, integral-exponential distributions of lake areas should be observed. At the first step, the parameters of these distributions were estimated for each sample. The minimum value of parameter e was taken for each of the samples, while parameter γ was found using the method of moments by numerical solution within a specially designed software module of the equation

$$
-\frac{1}{\gamma Ei(-\gamma \epsilon)} \exp(-\gamma \epsilon) = \overline{s},
$$

where \bar{s} is the average lake area.

Using the same module, the value of Pearson's criterion was calculated and compared with the critical value at a significance value of 0.99. The results of assessing the agreement between empirical and theoretical integral-exponential distributions using the Pearson criterion are given in Table 2. It can be seen that at the significance level of 0.99, the distribution of thermokarst lake areas within the main surface of thermokarst plains is consisted with the theoretical integral-exponential distribution at six out of eight

³ For small key sites, the probability of having more than one source is much greater.

District	Site number	Area, km ²	Sample size (number of lakes)		Maximum difference		
			on the main surface	on the surface of thermokarst depressions	negative	positive	Parameter p^*
Yamal Peninsula	20	450	86	254	-0.012	0.227	p < 0.005
	25	202	131	116	-0.092	0.098	p > 0.10
	30	4419	260	132	0.00	0.261	p < 0.001
Tazovsky Peninsula	19	207	87	105	0.00	0.359	p < 0.001
Kolyma Lowland	21	1157	252	172	0.00	0.276	p < 0.001
	22	2867	113	183	0.00	0.348	p < 0.001
	28	1343	125	117	-0.012	0.245	p < 0.005
Parapolsky Dol	40	670	350	175	0.00	0.231	p < 0.001

Table 1. **Comparison of empirical distributions of lake areas according to Smirnov's criterion**

* Difference in distributions is statistically significant at the 0.99 level in the case of $p < 0.01$.

sites. The distribution of thermokarst lake areas within thermokarst depressions also matches the theoretical integral-exponential distribution at six out of eight sites; in both cases, no correspondence is observed for site 30 (Penzhina Bay area).

In Fig. 4, graphs of thermokarst lake area distributions of the first and second types are provided; the difference between them and the correspondence of both graphs to the integral-exponential distribution are clearly seen.

Fig. 4. Graphs of thermokarst lake area distribution at sites 22 (*a***), 28 (***b***), 19 (***c***), 40 (***d***).**

1, *2* – empirical distribution of surface areas of type 1 and type 2 lakes, respectively; *3*, *4* – theoretical (integral-exponential) distributions of surface areas of type 1 and type 2, respectively.

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			Sample size	Criterion value			
District	Site number	Average lake area, m ²		Pearson	critical at the 0.99 level	ε , m ²	γ , km ⁻²
			Main surface				
Yamal Peninsula	20	120 564	86	7.678	6.635	2112	5.483
	25	54 647	131	11.034	11.341	487	3.155
	30	31 296	260	22.570	13.277	194	5.021
Tazovsky Peninsula	19	51774	87	5.537	9.210	473	3.271
Kolyma Lowland	21	300 137	252	5.413	15.086	6972	0.697
	22	638 574	113	4.870	11.341	7583	0.280
	28	568958	125	2.200	6.635	6898	0.316
Parapolsky Dol	40	27 7 5 3	350	13.143	13.277	217	5.917
			Surface of thermokarst depressions				
Yamal Peninsula	20	32 298	254	11.060	11.341	1307	7.573
	25	35 507	116	9.814	15.086	1519	5.416
	30	10 981	132	19.411	15.086	148	16.742
Tazovsky Peninsula	19	13 197	105	13.054	9.210	90	12.12
Kolyma Lowland	21	110 604	172	5.843	9.210	662	1.412
	22	115790	183	3.589	9.210	1037	1.457
	28	211 069	117	4.341	11.341	3274	0.899
Parapolsky Dol	40	10 169	175	4.093	15.086	163	18.804

Table 2. **Assessment of the agreement between empirical and theoretical integral-exponential distributions according to Pearson's criterion**

N o t e: γ is the estimate of the average density of the sources of stream channels; ε is the estimate of the initial size of the lakes.

Figure 5 shows the results of comparison of the parameters of both distributions for the same sites. The values of parameter γ , which, according to the model, reflects the average density of the sources of stream channels within the main surface and within thermokarst depressions are compared. Parameter e as the minimum value for the sample is more susceptible to the influence of random factors (for example, it depends on sample size). Therefore, it was not specially analyzed.

The correlation coefficient between the values of γ_1 and γ_2 is 0.95. The values of this parameter for lakes on the main surface γ_1 are always less than those for lakes in thermokarst depressions γ_2 .

DISCUSSION OF THE RESULTS OF COMPARATIVE ANALYSIS

Analysis of the obtained results allows us to conclude that area distribution of thermokarst lakes forming on the surface of thermokarst depressions differs from that of thermokarst lakes on the main surface, This is illustrated by the discrepancy between the graphs: the graph of area distribution of type 1 lakes lies to the right of the graph of area distribution of type 2 lakes for all the sites. This is a consequence of the fact that the proportion of small lakes within lake clusters in thermokarst depression is significantly higher than that for small lakes on the main surface. The differences in distributions are statisti-

Fig. 5. Relationship between estimates of the average density of sources of stream channels for different types of surfaces of thermokarst plains with fluvial erosion.

 γ_1 – main surface parameter value, γ_2 – thermokarst depression surface parameter value.

cally significant for all sites, except for site 25 (Yamal Peninsula).

Area distribution of thermokarst lakes forming on the main surface follows the integral-exponential distribution in 75% of the sites. As argued before [*Victorov et al., 2021*], this is a sign of dynamic equilibrium, some balance between the initiation and growth of the lakes, on one hand, and their drainage by erosional processes, on the other hand. This conclusion turns out to be valid provided that the model's assumptions are valid; in particular, the assumption about the ongoing process of the formation of new lakes. This assumption remains open to argument for the main surface, so the conclusion obtained is subject to further research and cannot be considered final.

The identified presence of integral-exponential distribution, although with other values of parameter γ for the lakes in thermokarst depressions, allows us to conclude that an analogous dynamic equilibrium is also established on this territory. This is also a consequence of the fulfillment of the model's assumptions, and in the given case the initiation of new lakes is explained by the appearance of new drained lake basins.

The difference in the values of parameter γ can be explained by the fact that within the area of thermokarst depressions, there are obviously more sources of stream channels owing to those that previously led to the formation of the existing thermokarst depressions; hence, parameter γ is higher. At the same time, at each site, the processes on the main surface and on the surface of thermokarst depressions occur within the same landscape – thermokarst plain with fluvial erosion and under uniform climatic conditions: the morphological parts of this landscape are linked by diverse interactions, including the movement of matter and energy. In our opinion, this circumstance explains good correlation between the values of parameter γ for lakes of the first and second types.

Analysis of the obtained results also allows us to conclude that the model of the development of the morphological pattern of thermokarst plains with fluvial erosion must consider the identified difference in the formation of thermokarst lakes on the main watershed surface and on the surface of thermokarst depressions.

CONCLUSIONS

This study allows us to make the following conclusions:

1. Comparison of area distributions of thermokarst lakes forming on the main surface of thermokarst plains with fluvial erosion and secondary lakes forming within the already existing thermokarst depressions shows the presence of statistically significant differences between them; therefore, the model of the development of the morphological pattern of thermokarst plains with fluvial erosion should be based on the existence of different conditions for the development of thermokarst lakes on the main surface of the plain and on the surface of thermokarst depressions, often forming clusters.

2. On each of the two types of surfaces within thermokarst plains with fluvial erosion, in a significant number of cases, integral-exponential distributions of the areas of thermokarst lakes are observed. Although they differ from one another, their existence can be interpreted as a sign of the establishment of dynamic equilibrium in the course of the development of thermokarst lakes, at least within the surface of thermokarst depressions.

3. Lake area distribution parameters for lakes on the main surface differ from those for lakes in thermokarst depressions; however, there is a good correlation between them.

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COMPARATIVE ANALYSIS OF AREA DISTRIBUTIONS FOR THERMOKARST LAKES WITHIN DIFFERENT TYPES

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