EARTH'S CRYOSPHERE
SCIENTIFIC JOURNAL

Kriosfera Zemli, 2021, vol. XXV, No. 1, pp. 40–47

METHODS OF CRYOSPHERIC RESEARCH

MODELING THE WAYS OF THE MORPHOLOGICAL PATTERN DEVELOPMENT FOR THERMOKARST PLAINS WITH FLUVIAL EROSION

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The research deals with four different hypotheses on the development of thermokarst plains with fluvial erosion based on the mathematical modeling of their morphological pattern. The models result from the mathematical morphology of landscapes, which broadly uses the random processes theory. The analysis revealed that each way of the development is characterized by a specific probabilistic distribution of sizes, areas of thermokarst lakes first of all. The empirical testing was done for 17 key sites with different environmental and permafrost conditions in Western and Eastern Siberia, and Canada. Our analysis revealed that in the majority of cases the areas of the thermokarst lakes within the homogenous sections of the thermokarst plains with fluvial erosion obey the integral exponential distribution. Hence, the model of the morphological pattern corresponding to the asynchronous start of thermokarst process is valid, and the increase in the size of the lakes is proportional to the heat loss density through the side surface. Thus, the morphological pattern of the vast areas of thermokarst plains with fluvial erosion is in a state of dynamic equilibrium, which should be taken into account when predicting its development and assessing natural risks.

Key words: *mathematical morphology of the landscapes, thermokarst plains with fluvial erosion, morphological pattern.*

INTRODUCTION

One of the most interesting objects in permafrost studies is the dynamics of landscapes developed in it [*Kirpotin et al., 2008; Kravtsova, Bystrova, 2009; Polishchuk, Polishchuk, 2013; Grosse et al., 2016*]. It is especially important to analyze the development of permafrost landscapes over a long time period. In recent studies, an attempt to solve a similar problem in relation to lacustrine-thermokarst plains was made [*Victorov et al., 2015*], but the dynamics of thermokarst plains with fluvial erosion has not been studied in detail.

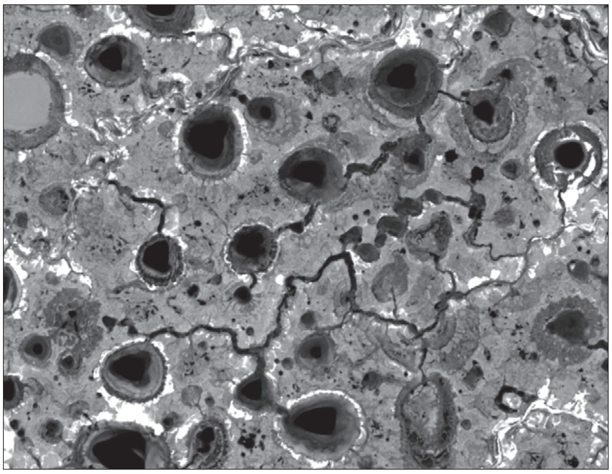

Fig. 1. A typical image of a site of thermokarst plain with fluvial erosion on a satellite image.

The purpose of the work is to study the regularities of changes in the morphological structure of thermokarst plains with fluvial erosion under different ways of its development.

The landscape of thermokarst plain with fluvial erosion is a slightly undulating flat surface with tundra or forest-tundra vegetation (cotton grass tundra, sedge-cotton grass tundra, etc.), with lakes, khasyreys and sparce erosion network (Fig. 1). The lakes are often rounded and scattered around the plain. Khasyreys are flat-bottomed peaty depressions with gentle slopes, also of isometric shape, occupied by grassland or swamp vegetation and, similarly to lakes, are disorderly located on the plain. Small residual lakes along the periphery and large lakes in the central part can remain inside the khasyrey. Secondary permafrost aggradation and development of frost mounds are possible within the khasyrey. It is generally accepted that khasyreys are formed as a result of drainage of thermokarst lakes, most often as a result of erosion.

The landscape of considered plains is under the complex influence of thermokarst, thermal abrasion and thermal erosion processes, which are manifested in the following elements:

– new primary thermokarst depressions appear;

– thermokarst depressions grow independently of each other like lakes due to thermal abrasion;

– at a random moment, the lake can be drained by erosion and turns into a khasyrey, while the growth of the basin stops due to the lack of water body.

Thus, the authors believe that the main reason for the lateral lake growth is the process of thermal abrasion. In particular, commonly rounded lake shape indicates this.

In the study, various versions of hypotheses for the development of the thermokarst plains with fluvial erosion on the basis of mathematical modeling of their morphological structure have been considered. Four hypotheses have been considered, each with its own mathematical model of development (models 1.0, 1.1, 2.0, 2.1). Hypotheses differs in the initial assumptions, which are analyzed in detail below (Table 1). The main differences between the initial assumptions are the ideas about the appearance of primary thermokarst depressions (synchronous or extended in time) and about the rate of growth (constant or changeable over time).

The model creation is based on the approaches of the mathematical morphology of landscapes [*Victorov, 2006; Kapralova, 2014*] using the theory of random processes.

METHODS OF THERMOKARST PLAINS DEVELOPMENT STUDYING

The model of the morphological structure of thermokarst plains with fluvial erosion describes an area with homogeneous environmental and permafrost conditions, under the modern climate. The model does not assume absolute homogeneity, but only statistical one. Model 1.0 is based on the following assumptions:

1. The process of thermokarst initiation occurred in a short period of time ("synchronous start"); it was probabilistic and went independently on non-overlapping sites, while the probability of depression formation on the test site depended only on the area of the test site[1].

2. The change in the radius of the resulting thermokarst depression is a random process; it occurs independently of other lakes, and its rate is proportional to the density of heat losses through the lateral submerged surface of the lake basin.

3. Growing lake can turn into a khasyrey in case of its drainage. The probability of that does not depend on other lakes. The growth of the lake stops after drinage.

Table 1. **Versions of considered models of the morphological pattern development**

Model	Assumptions	
	Start	Size growth
1.0	Synchronous	Proportional to the heat loss density
1.1	Asynchronous	Same
2.0	Synchronous	Quasi-uniform
2.1	Asynchronous	Same

4. The inception of erosional forms on non-intersecting areas are independent random events; the probability of the presence of initial erosional forms on the test site depends only on its area[2].

Let us clarify the second assumption. During the thermoabrasional interaction of the lake water mass with the edges not only the mechanical effect, but also the thermal effect of water on the edge plays an important role. Permafrost degrades while ground ice inside it melts out. The thermal effect works mainly on the underwater part of the lake. It seems natural to assume that the more heat of the lake's water mass is spent on ground ice melting at 1 square meter of the underwater side of the basin (in the article this value is conditionally named "density of heat losses"), the faster, other things being equal, the degradation of permafrost proceeds increasing the size of the lake. This simplified view was the basis for the second assumption.

For the case of a synchronous start, the following initial dependences obtained earlier in the mathematical morphology of the landscape remain to be valid [*Victorov, 2006*]:

– distribution of the radius of growing thermokarst lake in time t after the emergence of a given lake (logarithmically normal distribution)

$$f_0\,(x,t)=\frac{1}{\sqrt{2\pi}\sigma x\sqrt{t}}\exp\left(-\frac{\left(\ln x-at\right)^2}{2\sigma^2 t}\right), \quad (1)$$

where a, σ are parameters, t is time elapsed from the beginning of the process;

– distribution of the distance from the center of the growing lake to the head of nearest erosional form which could stop the lake growth with transformation into a khasyrey (Rayleigh distribution)

$$F(x)=1-\exp\left(-\pi\gamma x^2\right), \quad (2)$$

where γ is average density of the heads of erosional forms[3]; $P(k)=\frac{\lambda^k}{k!}\exp\left(-\lambda\right)$ – distribution of number

[1] In that case, the probability of formation of more than one depression is infinitely small value of a higher order than the probability of formation of one depression.

[2] In that case, the probability of occurrence of more than one initial erosional form is infinitely small of a higher order than the probability of occurrence of one initial erosional form.

[3] The average density of the heads of erosional forms is the average number of heads per the area unit (km^{-2}).

of primary depressions on the test site occurred at the beginning of thermokarst process (Poisson distribution).

Subsequently, the average density of the lakes decreases in accordance with the possibility of their transformation into khasyreys. Since the probability of lake's turning into a khasyrey does not depend on the location of the lake, it is easy to demonstrate that the distribution of the number of lakes and the number of khasyreys on the test site at any time remains to be Poissonian.

The distribution of the radii of khasyreys at an arbitrary moment is determined by the distance to the nearest head of the erosional form which will stop the growth, and, hence, the distribution of the area of the khasyreys under a long-term development corresponds to an exponential distribution [*Victorov et al., 2016*].

The distribution of the radii of thermokarst lakes at an arbitrary point in time is determined by the distribution of the corresponding radius under conditions of free growth, but given that the lake will not become a khasyrey, that is, the distance to the head of erosional form will be greater than the lake radius

$$f_l(x,t)=\frac{f_0(x,t)\exp\left(-\pi\gamma x^2\right)}{\int_0^{+\infty} f_0(u,t)\exp\left(-\pi\gamma u^2\right)du}. \quad (3)$$

Using the expression for free lake growth (1) and simplifying it due to the identical terms in the numerator and denominator, which depend only on time, we find that under long-term development ($t \to +\infty$) the density of the lake radius distribution tends to the limiting distribution

$$f_l(x,\infty)=\frac{x^{a/\sigma^2-1}\exp\left(-\pi\gamma x^2\right)}{\int_0^{+\infty} x^{a/\sigma^2-1}\exp\left(-\pi\gamma x^2\right)dx},$$

which is χ distribution. Hence, taking into account the circular shape of the lake, we obtain that the limiting distribution for the lake area is the gamma distribution

$$f_{sl}(x,\infty)=\frac{\gamma^{a/2\sigma^2}}{\Gamma\left(a/\left(2\sigma^2\right)\right)}x^{a/2\sigma^2-1}\exp\left(-\gamma x\right),$$

where $\Gamma(x)$ is the gamma function.

Thus, based on the assumptions of the model, the main expressions describing the features of the morphological structure of the thermokarst plains with fluvial erosion and its dynamics have been obtained in the version 1.0 of the model.

The second version of the model (1.1) differs from the first in the assumption 1a ("asynchronous start"):

1a. Emergence of the primary thermokarst depressions (initial point) for non-intersecting time intervals and at non-overlapping areas is independent random events; the probability of depression formation depends only on the duration of the time interval and the size of the site[4].

Model 1.1 introduces an additional fifth assumption:

5. The emergence of primary thermokarst depressions does not occur in the area of existing thermokarst lakes.

For the case of asynchronous start, two initial dependences obtained earlier ((1) and (2)) also remain valid.

Analysis of the model's assumptions allows one to obtain significant conclusions about the process dynamics. The formation of primary thermokarst depressions in a lake-free area without erosion is described by a Poisson process, as has been demonstrated earlier (for example, [*Victorov 2006; Victorov et al., 2016*]). However, if we take into account the second assumption of the model (primary depressions are formed only outside the lakes area), then the density of primary depressions formation is variable

$$\lambda_1(t)=\lambda\left[1-P_l(t)\right],$$

where $P_l(t)$ is the area percentage. The function $P_l(t)$, as shown earlier (for example, [*Victorov, 2006*]), is related with the process parameters through the dependence:

$$P_l(t)=1-\exp\left[-\tau(t)s(t)\right], \quad (4)$$

where $s(t)$ is average lake area at a point of time t; $\tau(t)$ is average number of lakes per unit area at a time t.

The distribution of the number of lakes, as well as khasyreys, remains Poissonian, since the probability of turning into khasyreys does not depend on the location of the lake.

The distribution density of the lake radii at the moment of time t will be equal to the ratio of the number of lakes with a given radius (taking into account the different times of their initiation and the probability of preservation without turning into khasyreys) to the total number of lakes, and after simplification it will be:

$$f(x,t)=\exp\left(-\pi\gamma x^2\right)\int_0^t\left[1-P_l(u)\right]f_0\left(x,t-u\right)du\times$$
$$\times\left[\int_0^t\left[1-P_l(u)\right]\int_0^{+\infty}\exp\left(-\pi\gamma x^2\right)f_0\left(x,t-u\right)dxdu\right]^{-1}. \quad (5)$$

[4] In that case, for the small areas and short time intervals, the probability of formation of several depressions is much less than of a single one.

The model allows one to obtain an expression describing the dynamics of the lake area percentage. Using the above-mentioned law of the lake radius distribution to determine the value of the average area of the lake, and the expression for the average density of the lake location and the expression (4), after simplification and taking the logarithm, we can obtain the integral equation [*Victorov, 2006*]:

$$\ln\left[1-P_l(t)\right]=-\pi\lambda\int_0^t\left[1-P_l(u)\right]\times$$
$$\times\int_0^{+\infty}x^2\exp\left(-\pi\gamma x^2\right)f_0\left(x,t-u\right)dxdu, \qquad (6)$$

the solution of which is the function of lake area percentage.

Of particular interest is *long-term behavior of each process*, since quite often the researcher is faced-up with longstanding processes. Using the integral equation (6), it can be demonstrated that if the integral of lake area percentage

$$I=\int_0^{+\infty}\int_0^{+\infty}x^2\exp\left(-\pi\gamma x^2\right)f_0(x,u)dxdu,$$

converges and solution of the equation[5]

$$\ln\left[1-P_l^*\right]=-\lambda\pi\left[1-P_l^*\right]I$$

does not exceed $(1-e^{-1})$, that is 0.63, then there exists a limit of the function $P_l(t)$ with $t\to+\infty$, and it is equal to the solution of that equation $\left(P_l^*\right)$. The proof is based on the construction of a pair of step functions bounding the function $P_l(t)$ from above and below. Using the equation (6), it can be demonstrated that under the above condition, both step functions converge to a single limit. According to the well-known theorem, the function $P_l(t)$ must also have the same limit.

Under those conditions, ensuring the existence of the limiting value of the lake area $\left(P_l^*\right)$, there is also a limiting distribution of the radii of the lakes with $t\to+\infty$. Using the expression (1) for the distribution density of the lake radius under free growth and calculating the upper integral of the expression (5) at $t\to+\infty$ as the value of the Laplace transformation [*Victorov, 2006*], we obtain that the distribution of the lake area with the area of the primary depression ε will correspond to the expression

$$f_{sl}(x,\infty)=-\frac{2}{x\mathrm{Ei}(-\gamma\varepsilon)}\exp\left(-\gamma x\right),\quad x\ge\varepsilon,$$

where $\mathrm{Ei}(-x)$ is an integral exponential function. Lake area distribution can be called “integral-exponential”. Finally, it follows from the obtained result [*Victorov, 2005*] that in that case there is a limiting value for the average number of lakes per unit area.

Thus, according to the model 1.1, after a long $(t\to+\infty)$ time, a dynamic equilibrium is established in the processes of generation of the thermokarst lakes (initial formation of primary thermokarst lakes) and their transformation into khasyreys.

The third version of the model (model 2.0) is based on the assumption of a synchronous start, as well as on the assumption that the lake growth occurs quasi-uniformly. The basis for that assumption is empirical data [*Burn, Smith, 1990; Smith et al., 2005*]. In that case, the second assumption is changed:

2a. The change in the radius of the resulting thermokarst depression is a random process; the change per unit of time is an independent, equally distributed random variable.

For that option, the first initial dependence is altered [*Victorov et al., 2015*], i.e. the radius distribution of a freely growing thermokarst lakes obeys the normal distribution

$$f_0\,(x,t)=\frac{1}{\sqrt{2\pi}\sigma\sqrt{t}}\exp\left(-\frac{(x-at)^2}{2\sigma^2t}\right), \qquad (7)$$

where a, σ are the distribution parameters, t is time elapsed since the beginning of the process.

The distribution of lake radii is determined by the expression (3), but with a free growth function corresponding to quasi-uniform growth (7). Calculating the integral and simplifying, we find that at any moment of time the lake radii must obey the normal law with following density of distribution:

$$f_l\,(x,t)=\frac{1}{\sqrt{2\pi}\sigma_l(t)}\exp\left(-\frac{\left[x-a_l(t)\right]^2}{2\sigma_l^2(t)}\right),$$

were $a_l(t)=\dfrac{at}{2\pi\gamma\sigma^2t+1}$, $\sigma_l(t)=\sigma\sqrt{\dfrac{t}{2\pi\gamma\sigma^2t+1}}$.

Passing to the limit, we find that with $t\to\infty$ the distribution of lake radii in model 2.0 is also close to the normal one.

In the fourth version of the model (2.1) the first assumption is replaced by 1a (asynchronous start), and, as in model 1.1, an additional fifth assumption appears about the impossibility of new thermokarst depressions generation within existing lakes. For that option, two initial dependencies are preserved: the distribution of the radius of a freely growing thermokarst lake in time t after the emergence of that lake, which obeys a normal distribution, and the Rayleigh distribution of the distance to the heads of erosional forms.

Analysis of the model 2.1, as in the model 1.1, leads to the conclusion about the Poisson distribu-

[5] It can be demonstrated that the equation always has a solution, and a sole one.

Table 2. **Pattern of lake size distribution for different models**

Model	The type of statistical distribution of lake sizes
1.0	Gamma distribution
1.1	Integral-exponential distribution
2.0	Normal distribution (average radius)
2.1	Gamma distribution

tion of both lakes and khasyreys. Performing similar analysis under longer development time, we also find that under general conditions, a dynamic equilibrium is established between the processes of generation of the thermokarst lakes and their transformation into khasyreys, and we also find that the exponential distribution of the areas of khasyreys is valid.

The distribution density of the radius of lakes under long-term development can be determined using the expression for the limiting distribution of the radius in the model 1.1, if we replace the density function of the radius distribution with free growth by (7)

$$f_l\ (x,\infty)=\sqrt{\gamma}\exp\left(-\pi\gamma x^2\right),\quad x\geq 0,$$

and, therefore, the area of lakes obeys the gamma distribution in that model, but with a fixed value of the shape parameter equal to 0.5

$$f_{sl}(x,\infty)=\sqrt{\frac{\gamma}{\pi}}\frac{\exp\left(-\gamma x\right)}{\sqrt{x}}.$$

Analysis of hypotheses about the development of thermokarst plains with fluvial erosion reveal that the ways differ in the distribution patterns of the lake areas (Table 2).

EMPIRICAL VERIFICATION AND RESULTS OF RESEARCH OF THERMOKARST PLAINS DEVELOPMENT

Empirical verification included analysis of the correspondence of thermokarst lakes areas distribution to different types of distribution. The results of comparing the correspondence of the distribution of the number of centers of lakes and khasyreys on a randomly selected site to Poisson's law have already been published and there is a good agreement [*Victorov et al., 2019a*], the same applies to the distribution of areas of khasyreys [*Victorov et al., 2019b*].

For empirical verification of thermokarst lakes areas distribution the sites with different environmental and permafrost conditions have been selected (Fig. 2, Table 3). The following satellite imagery were used: Corona (2–10 m/pix, 1965–1976); medium resolution Sentinel 2A images 2018–2019, 10 m/pix; high resolution images 0.5–1.5 m/pix (SPOT 6, 7, WorldView 2, June–August 2013–2019), both specially ordered and obtained from open sources (Google, Bing, Yandex mosaics). The latest surveys from 2013–2019 are named *term 2*, and the data from Corona imagery is named *term 1*.

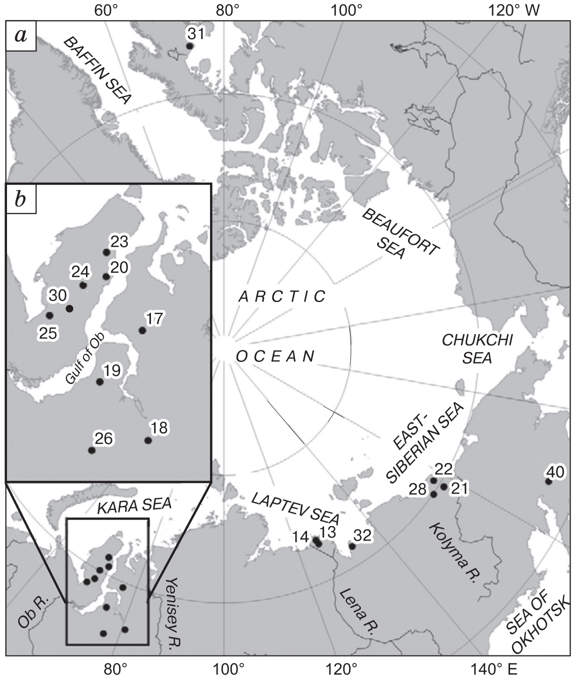

Fig. 2. Layout of key sites of thermokarst plains with fluvial erosion:

a – general scheme, *b* – enlarged scheme of the Yamal-Gydan region.

The boundaries of thermokarst depressions were identified both by an automated method – based on the ArcGIS and QGIS software modules according to the spectral brightness of the image, – and in the expert mode.

The verification of the correspondence of the theoretically obtained distributions to the empirical data was carried out for most distributions by means of a software package for statistical analysis using the Pearson test based on a well-known technique (see, for example, [*Kramer, 1970*]), in compliance with the conditions of its application. For the integral-exponential distribution, a special module to calculate the value of the Pearson criterion has been created (by P.V. Berezin). In that case, the minimum value of the sample was taken as estimate of the ε-parameter, and the γ-value was found by the method of moments by numerical solution within the framework of the same program module of the equation

$$-\frac{1}{\gamma \mathrm{Ei}(-\gamma\varepsilon)}\exp\left(-\gamma\varepsilon\right)=\overline{s_l},$$

where $\overline{s_l}$ is the average area of a lake.

Empirical verification for the model of thermokarst plains with fluvial erosion has given the following main results. The empirical data obtained for the areas of thermokarst lakes for the key sites of thermokarst plains with fluvial erosion included samples from 62 to 598 elements (Table 4).

Table 3. **Characteristics of test sites (fragment)**

Site number (see Fig. 2)	Site characteristics	Mean annual permafrost temperature, °C	Permafrost thickness, m
13	Accumulative type of relief, flat or gently sloping alluvial-marine plain of the Q_{III}–H age with sections of lacustrine boggy plains of the Late Holocene age, with numerous thermokarst and talik lakes, pingos, boggy depressions, water tracks. Sediments: sandy loams with interlayers of sands, plant detritus and peat at the areas of thermokarst lakes, lacustrine and boggy clayey silts. Outside the areas of lake distribution – alluvial-marine deposits represented by sands with interlayers of sandy loam and peat. Continuous permafrost distribution	–9...–11	300–400
22	Accumulative relief type. Flat lacustrine boggy (alas) plain, created by the joint activity of thermokarst and lacustrine-paludal processes, corresponds to the final stage of destruction of the Yedoma plain by thermokarst. Extensive confluent lacustrine basins of different stages of alas formation. The time of the formation of the relief is from the Late Pleistocene to early Holocene. Sediments: loess-like lacustrine-alluvial gray clayey silts, less often sandy silts with peat interlayers and lenses. Continuous permafrost distribution	–9...–11	300–500
23	Accumulative relief type. II alluvial-marine terrace of the Ob Bay of the Q_{III} age. Flat, swampy plain with numerous lakes. Sediments: sands with thin layers of sandy loam and loam. Continuous permafrost distribution	–7...–9	200–300
26	Erosion-accumulative relief type, the third lacustrine-alluvial terrace of the Pur River of the Q_{III} age. A flat plain, partly swampy with numerous drained lake basins. Frost mounds and thermokarst subsidence are widely developed. Sediments: the basal part is dominated by fine and medium-grained sands with inclusions of pebbles and gravel with oblique bedding. The middle part of the section (floodplain facies) is represented by silty sands and fine sands, often peaty with interlayers of sandy loam and loam. The lacustrine facies are composed of loams, less often clays and silts with interlayers of fine sand. Sporadic permafrost distribution	– 2...–0.5	0–15

For term 2, in 10 sites out of 17 (59 %), the empirical distributions correspond to the integral-exponential one (Fig. 3), which corresponds to the asynchronous start model 1.1. For term 1, the compliance is observed in 5 out of 11 sites (45 %). At the same time, in 8 out of 17 sites, the distribution of lake areas corresponds to the lognormal distribution, typical for lacustrine-thermokarst plains. In 3 out of 17 sites, the empirical distributions correspond to the gamma distribution; all the same samples correspond to the lognormal distribution, which corresponds to the synchronous start model [*Victorov, 2006; Victorov et al., 2016, 2019b*]. The closeness to the lognormal distribution, as in the case of the khasyreys, is fully explained by two factors:

– the thermokarst plains with fluvial erosion at the initial stages were lacustrine-thermokarst plains, since the probability of the drainage of lakes with their initially limited sizes was small, and those plains are characterized by a lognormal distribution of lake areas;

– integral-exponential distribution is the limiting distribution with $t \to +\infty$, and the time elapsed since the beginning of the thermokarst process is, although long, but finite one.

Interestingly, for typical lacustrine-thermokarst plains, only the lognormal distribution is valid, and no correspondence with the gamma distribution is observed. The distribution of the average radii of the lakes does not correspond to the normal one in any area. Finally, in 5 sites for term 2, the distributions do not correspond to any of the studied species.

Thus, on the whole, the empirical verification forces us to conclude that the situation of asynchronous start described by the model 1.1 for thermokarst plains with fluvial erosion is realized in nature over a sufficient number of territories. The theoretically obtained patterns – the integral-exponential distribution of lake areas – are confirmed empirically on a significant number of key areas.

At the same time, signs of a synchronous start described by the model 1.0 present only in 17 % of the sites (they have a gamma distribution of lake areas), while in 11 % a similarity with the integral-exponential distribution simultaneously is observed.

Analysis of the data reveals that the hypothesis about the possibility of the development of erosional-thermokarst plains with a synchronous start apparently does not correspond to reality, since no sites was found with the distribution of the average radii of the lakes close to the normal. At the same time, signs of the development of the considered plains with the validity of the hypothesis of quasi-uniform growth and asynchronous start (model 2.1), are observed in the form of a gamma distribution of the area of lakes with a shape parameter value equal to 0.5 only in 1 site out of 17.

Table 4. **Correspondence of empirical and theoretical distributions of thermokarst lakes areas**

Site	Term*	Sample size	Distribution			
			Lognormal	Gamma	Normal**	Integral-exponential
13	2	581	0.000	0.000	0.000	0.000
13	1	598	0.000	0.000	0.000	0.000
14	2	209	**0.014**	**0.017**	0.000	**0.022**
17	2	232	0.005	0.000	0.000	0.002
18	2	62	**0.160**	**0.018**	0.000	**0.086**
19	2	161	**0.017**	0.000	0.000	**0.213**
19	1	160	**0.091**	0.000	0.000	**0.394**
20	2	318	0.007	0.000	0.000	0.000
20	1	359	0.000	0.000	0.000	0.000
21	2	405	0.000	0.000	0.000	**0.109**
21	1	339	**0.010**	0.000	0.000	0.004
22	2	244	0.000	0.000	0.000	**0.014**
22	1	337	0.000	0.000	0.000	**0.641**
23	2	257	**0.044**	0.000	0.000	**0.220**
24	2	346	0.004	0.000	0.000	**0.663**
24	1	376	0.001	0.000	0.000	**0.024**
25	2	278	**0.225**	0.000	0.000	0.000
25	1	281	**0.265**	0.000	0.000	0.000
26	2	500	0.008	0.000	0.000	0.001
28	2	264	**0.310**	0.000	0.000	**0.053**
28	1	267	**0.122**	0.000	0.000	**0.085**
30	2	519	**0.322**	0.000	0.000	**0.245**
30	1	519	**0.710**	0.000	0.000	**0.023**
31	2	74	0.000	**0.015**	0.000	0.005
31	1	70	0.000	0.000	**0.018**	0.001
32	2	430	0.000	0.000	0.000	0.000
32	1	439	0.000	0.000	0.000	0.000
40	2	535	0.001	0.000	0.000	**0.122**

Note: the table presents p-values (the probability of exceeding the actual value of the chi-squared criterion); empirical data do not contradict theoretical ones at the significance level of 0.99, if $p > 0.01$ (in bold).

* Term 1 – 1965–1976 (Corona imagery); term 2 – 2013–2019.

** For mean radii of lakes.

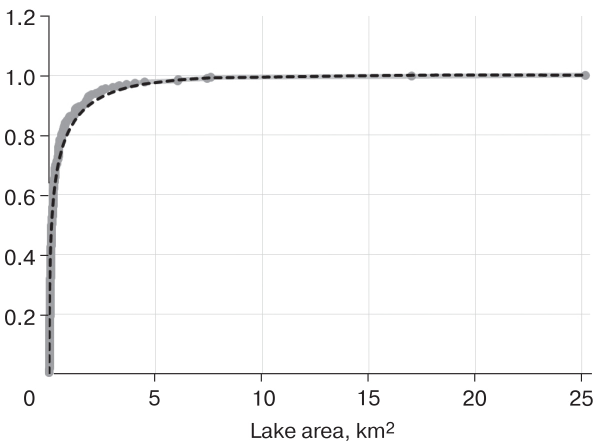

Fig. 3. An example of the correspondence of the empirical distribution of lake areas (solid line) to the cumulative-exponential distribution (dashed line), Site 28.

Thus, the analysis of the data allows us to reveal that most of the distributions can be explained using the hypothesis of lake growth, although determined by the influence of many random factors, but proportional to the density of heat losses through the lateral surface of the lake (models 1.0 and 1.1). That is consistent with the results of the analysis of lacustrine-thermokarst plains [*Victorov, 2006; Victorov et al., 2015, 2016*]. At the same time, at first glance, there is inconsistency in other obtained results, – the analysis of lacustrine-thermokarst plains indicated the hypothesis of synchronous start. In more than a half of the cases there are signs of an asynchronous start (model 1.1). This can be partly explained by the favorableness of the khasyreys area for the development of a large number of secondary lakes.

Let us emphasize that the determination of the lake area percentage indicates the fulfillment of the above-mentioned condition, for which the tendency of the parameters to the limiting values in the model 1.1 has been proved. For all areas, the lake area percentage is within the range of 0.01–0.50, reaching a value of 0.66 only in site 24.

Inconsistency with any distributions in 5 sites can be caused by the transient processes under climatic changes. The study has demonstrated that exactly in 2 out of 3 of those sites, which are part of the five and at the same time have two survey dates, the distribution of lake areas for term 1 and term 2 differs significantly (according to Smirnov's criterion). At the same time, such differences are observed in 4 areas among all investigated.

This analysis certainly does not claim to be exhaustively reliable, and it will be clarified. The performed analysis is limited primarily due to the assumption of insignificant climate changes. At the same time, such analysis should have been carried out primarily as an analysis of the situation with the simplest conditions.

CONCLUSIONS

1. In homogeneous areas of thermokarst plains with fluvial erosion in a variety of environmental and permafrost conditions, the model of the development of the morphological structure is valid in most cases. It corresponds to an asynchronous start and growth of lakes size proportional to the density of heat losses through the lateral surface.

2. In homogeneous areas of thermokarst plains with fluvial erosion under different environmental and permafrost conditions the integral-exponential law of thermokarst lakes areas distribution is valid in most cases.

3. Analysis of the development of thermokarst plains with fluvial erosion for the period from 1965 to 2019 demonstrated that with all the observed changes, the plain's structure does not change in a significant number of areas, but is in a state of dynamic equilibrium, despite the widely discussed impact of climate change. That should be taken into account when forecasting lake development and assessing natural risks.

The study has been carried out with the financial support of the Russian Science Foundation, project No. 18-17-00226.

References

Burn, C.R., Smith, M.W., 1990. Development of thermokarst lakes during the Holocene at sites Near Mayo, Yukon Territory. Permafrost and Periglacial Processes I, 161–176.

Grosse, G., Jones, B.M., Nitze, I., et al., 2016. Massive thermokarst lake area loss in continuous ice-rich permafrost of the northern Seward Peninsula, Northwestern Alaska, 1949–2015. In: XI International Conference on Permafrost: Abstracts (Potsdam, 20–24 June 2016). Potsdam, Germany, pp. 739–740.

Kapralova, V.N., 2014. Regularities of the development of thermokarst processes within the lake-thermokarst plains (based on the approaches of the mathematical morphology of landscapes). Abstract of PhD thesis. Moscow, 24 pp. (in Russian).

Kirpotin, S.N., Polishchuk, Yu.M., Bryksina, N.A., 2008. The dynamics of the areas of thermokarst lakes in continuous and discontinuous permafrost zones of Western Siberia under global warming. Vestnik TSU [Bulletin of TSU], No. 311, 185–189.

Kramer, G., 1970. Mathematical Methods of Statistics. Mir, Moscow, 648 pp. (in Russian).

Kravtsova, V.I., Bystrova, A.G., 2009. Changes in thermokarst lake size in different regions of Russia for the last 30 years. Kriosfera Zemli [Earth's Cryosphere], XIII (2), 16–26.

Polishchuk, V.Yu., Polishchuk, Yu.M., 2013. Geosimulation modeling of fields of thermokarst lakes in permafrost zones. Ural State University, Khanty-Mansiysk, 129 pp. (in Russian).

Smith, L.C., Sheng, Y., Macdonald, G.M., Hinzman, L.D., 2005. Disappearing Arctic Lakes. Science 308 (3), p. 14.

Victorov, A.S., 2005. Mathematical models of thermokarst erosion plains. In: GIS and Spatial Analysis. Proc. of IAMG, Toronto, Canada, printed by China University of Geosciences Printing House, vol. I, pp. 62–67.

Victorov, A.S., 2006. The main problems of the mathematical morphology of the landscape. Nauka, Moscow, 252 pp. (in Russian).

Victorov, A.S., Kapralova, V.N., Orlov, T.V., et al., 2015. An analysis of the morphological structure development of the thermokarst-lake plains on the base of the mathematical model. Geomorfologiya [Geomorphology], No. 3, 3–13.

Victorov, A.S., Kapralova, V.N., Orlovm, T.V., et al., 2016. Mathematical morphology of cryolithozone landscapes. RUDN, Moscow, 232 pp. (in Russian).

Victorov, A.S., Orlov, T.V., Sadkov, S.A., Trapeznikova, O.N., 2019a. Remote assessment of natural hazards on the base of the mathematical morphology of landscape. Geoekologiya. Inzhenernaya geologiya. Gidrogeologiya. Geokriologiya [Geoecology. Engineering geology. Hydrogeology. Geocryology], No. 5, 61–73.

Victorov, A.S., Orlov, T.V., Arkhipova, M.V., 2019b. Laws for size distribution of khasyreis. Doklady Earth Sciences, 488 (2), 1253–1255.

Received March 13, 2020
Revised version received November 4, 2020
Accepted November 22, 2020