*Earth`s Cryosphere*, 2015, vol. XIX, No. 3, pp. 58–63 **http://earthcryosphere.ru/**

# PROSPECTS OF ATMOSPHERIC HEAT UTILIZATION IN PERMAFROST TERRITORIES

**G.Z. Perlshtein, G.S. Tipenko, A.V. Levashov***

*Sergeev Institute of Environmental Geoscience, RAS,*
*13, build. 2 Ulansky per., Moscow, 101000, Russia; cryo2@yandex.ru*
*[*] KBC Process Technology Ltd, 47/2 Leningradsky av., Moscow, Russia*

The performance of a film water heater is analyzed using a theoretical approach. Calculation research is accomplished on the basis of generally accepted design equations. Direct dependence has been established between the water heating rate and the temperature on its surface. This important conclusion allows the numerical modelling procedures to be essentially simplified. The calculations made show broad prospects for the use of this simple water heater in the Russian permafrost territory.

*Film water heater, solar radiation, convective heat transfer, Prandtl and Grashof criteria, economic effect*

## INTRODUCTION

The territory of the Russian permafrost zone has huge renewable sources of atmospheric heat. In summer, in the sunny day hours, the intensity of short-wave solar radiation in the major part of the permafrost zone exceeds 1 kW/m$^2$. However, as the temperature of the heat recipient grows, the amount of waste of energy coming to the atmosphere increases, too. This accounts for the challenges of developing this powerful source of natural heat.

Solar energy collectors are widespread in the world; their main components are an absorbing metal panel (usually made of aluminum, copper, brass, or stainless steel), glass, insulation material (normally consisting of foamed plastic), and enamel coating. However, their cost varies from 70 to 200 USD per 1 square meter [*Butuzov, 2000*]. As a rule, the payback period exceeds 30 years. It is clear that, in the current economic situation in Russia, such projects will not find investors for such a long-term perspective. At the same time, large amounts of heat may be collected in the summer period with the help of solar water heaters made of film sleeves.

The studies relating to industrial application of solar water heaters in the economy of the permafrost zone were conducted in the late 60ies on testing dredge sites of the northeast USSR [*Goltman et al., 1970*]. At that time, the theory of heat exchange in the "day surface–atmosphere" system was definitely insufficiently developed. In particular, there was no correct calculation of the growth in heat losses as the water temperature increased, and analysis of the results obtained was based on the so-called uptaking capacity of solar radiation, which could not be evaluated in advance. Perhaps, it was due to the incomplete understanding of the physical nature of the process that the consumption of water per area unit was assumed to be too large in the experiments – about 0.1 m$^3$/(m$^2$·h). As a result, the water temperature rose insignificantly, and further experiments stopped.

The meteorological observations in the continental regions of Siberia and the Russian Far East recorded the absolute maximum values of the soil surface temperature to be within the range of 51–59 °C. The use of film sleeves allows eliminating evaporation and essential reduction of heat losses due to convective heat exchange with air, which may result in significant contribution to the rise of the water temperature.

## THE OPERATING PRINCIPLE OF A FILM WATER HEATER

The schematic layout of one of the possible variants of a simple solar water heater is shown in Fig. 1. Its main components are: I – a solid heat insulation base with a black surface ($x < 0$); II – water in the sleeve made of transparent film ($0 < x < h$); III – an air insulator limited by transparent film from above ($h < x < s$).

In accordance with the Beer–Lambert–Bouguer law [*Mikheyev and Mikheyeva, 1977*], attenuation of the flux density $Q(h)$ in a layer of water with the height of $h$ is described by the formula

$$Q(h) = Q_0 \exp(-\mu h), \qquad (1)$$

where $Q_0$ is the density of the incoming beam of light. According to F.E. Are [*Pavlov, 1979*], for lacustrine water the value of coefficient μ is close to 0.7. With μ = 0.7, a one-centimeter layer of water passes more than 99 % of solar radiation.

In the proposed design, there is a source of heat at the bottom of a film sleeve with a transparent top $Q_s(1 - A)$, where $Q_s$ is short-wave solar radiation, and $A$ is albedo. The main heating of water occurs from the bottom, resulting in free convection. Heat trans-

**Copyright © 2015 G.Z. Perlshtein, G.S. Tipenko, A.V. Levashov, All rights reserved.**

fer in the air insulator takes place due to radiation and free convection. More or less correct metering of the process is possible only on the basis of joint solution of a system of Navier–Stokes equations for two regions with drastically differing characteristics of water and of the air layer. Calculations of the temperature field of the liquid were made using a special software program, which allowed taking into account the details of heat transfer in the approximation of Oberbeck–Boussinesq [*Kutateladze, 1970*].

In setting the boundary conditions for the surface of the air insulator, we used modern assumptions relating to heat exchange of the day surface of the Earth with atmosphere [*Budyko, 1956; Olovin, 1971; Perlshtein, 2002; Perlshtein et al., 2005; Pavlov et al., 2010*]. All the factors of "external" heat exchange are divided into regional and microclimatic ones. Heat flux with the density of $q_0$ [W/m$^2$] comes to the reference surface (with the temperature 0 °C), which depends only on the regional climatic characteristics. Heat losses from the surface increase as its temperature $T(s)$ grows to be equal to $(\alpha_a + \alpha_l)T(s)$.

In the absence of evaporation, the heat flux $q$, directed from the insulator surface ($x = s$) to the sleeve containing water ($x = h$), is expressed by the following dependences:

$$q = q_0 - (\alpha_a + \alpha_l)T(s); \qquad (2)$$

$$q_0 = Q_s - AQ_s + I_a - I_0 - \alpha_a T_a; \qquad (3)$$

$$\alpha_l(T) = \sigma\delta\left[\frac{(T(s)+273)^4 - 273^4}{T(s)}\right]. \qquad (4)$$

Here $T_a$, $T(s)$ are temperatures of air and of the air insulator's surface, accordingly, °C; $Q_s$ is total short-wave radiation, W/m$^2$; $A$ – albedo, units; $I_a$ is long-wave radiation of the atmosphere, W/m$^2$; $\alpha_a$ is coefficient of convective heat exchange with air, W/(m$^2$·K); σ – the Stefan–Boltzmann constant, equal to 5.67·10$^{-8}$ W/(m$^2$·K$^4$); δ is emissivity*, for natural surface usually close to 0.90–0.95; $I_0$ is surface radiation at 0 °C; $\alpha_l$ is the approximation coefficient of the radiation law, W/(m$^2$·K). Depending on the wind velocity, coefficient $\alpha_a$ was calculated by the empirical formula of Jürgens [*Kurtener and Chudnovsky, 1969*]:

$$\alpha_a = \begin{cases} 6.16 + 4.19\,u, & 0 < u < 5, \\ 7.56\,u^{0.78}, & 5 < u < 30, \end{cases}$$

where $u$ is wind velocity, m/s.

In simulating water heating in the warm period, the upper boundary conditions were set based on the presented theoretical assumptions. Climatic characteristics were used, typical of the day hours in the continental part of the permafrost territory of Russia: $T_a$ = 15 °C; $Q_s$ = 440 W/m$^2$; $I_a$ = 300 W/m$^2$; $u$ = 2.5–3.0 m/s; $\alpha_a$ = 20 W/(m$^2$·K).

Numerical modeling of the heat processes taking place in a water-containing sleeve without a layer of air insulation showed intense free convection (Fig. 2) and increase in effective thermal conductivity caused by it.

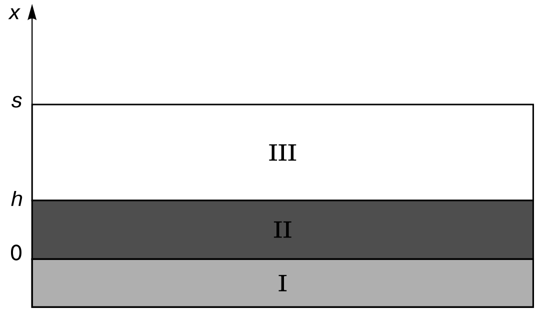


**Fig. 1. A schematic design of a film water heater.**
See the explanations in the text.

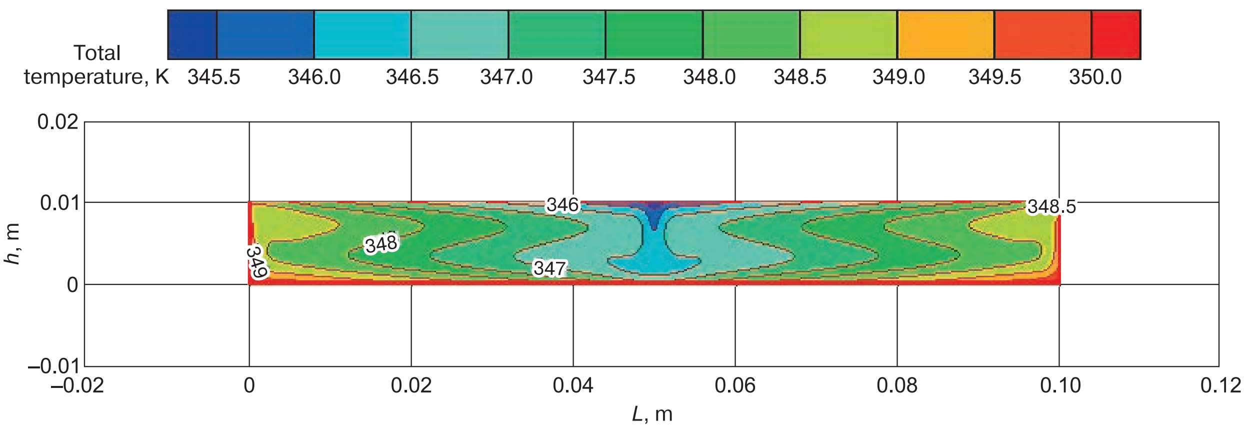


**Fig. 2. The temperature field (K) of an elementary convective cell ($L$ – length, $h$ – height).**

* In addition to the term "emissivity", equivalent notions are used: the grayness value and the degree of blackness.

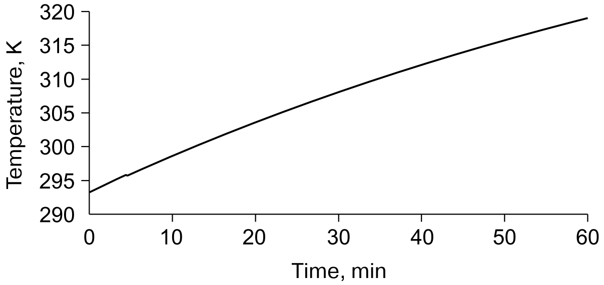


**Fig. 3. Estimated dynamics of water temperature.**

The important results of the numerical calculations also include the high final water temperature (~78 °C) and the character of its change in time. At the initial moment, the heating rate is 0.7 °C/min, by the end of the first hour it drops nearly 2.5 times (Fig. 3). The heating rate proves to be practically uniformly slow.

Extrapolating the data obtained, we find that the water temperature will reach 70 °C approximately in 4 hours 20 minutes. It is to be noted that in the process of water heating the temperature difference in the layer of water did not exceed 1.5 °C.

Unfortunately, development of a complete thermal physical model proceeds very slowly on standard computers. The calculation time required for obtaining the results shown on Fig. 3 was more than 50 hours.

## THE NUMERICAL MODEL OF THE PROCESS

It is much more complicated to describe heat exchange in an area of calculations covering the entire solar water heater (Fig. 1). The conditions of connections on the boundaries of the areas with coordinates 0, $h$ and $s$ look as follows:

$$x = 0: \quad -\lambda_{w,ef} \left.\frac{\partial T_w}{\partial x}\right|_{x=0} = Q_s(1-A)\,; \tag{5}$$

$$x = h: \quad \lambda_{w,ef} \left.\frac{\partial T_w}{\partial x}\right|_{x=h} = \frac{T(s)-T(h)}{R} + \delta^{*}\sigma\Big[\big(T(s)+273\big)^4 - \big(T(h)+273\big)^4\Big]; \tag{6}$$

$$x = s: \quad \delta^{*}\sigma\big(T(h)+273\big)^4 + \frac{T(h)}{R} = \sigma(\delta^{*}+\delta)\big(T(s)+273\big)^4 + \frac{T(s)}{R} + \alpha_a T(s) - \alpha_a T_a - I_a\,, \tag{7}$$

where $R$ is thermal resistance to air insulation, $(m^2 \cdot K)/W$; $\lambda_{w,ef}$ is coefficient of effective (considering free convection) thermal conductivity of water; $T$ is temperature, °C; $\delta^{*}$ is reduced emissivity of the radiating surfaces (upper and lower) of the air insulator; indices $a$, $w$, $i$ refer to atmosphere, water, and air insulator, respectively.

Conditions (6) and (7), in particular, indicate that throughout the entire heating process each value of temperature $T(h)$ is related to a strictly determined value $T(s)$, and conversely. The reduced emissivity takes into account the effect of reflection and absorption of heat rays between two parallel surfaces and is described by Christiansen's formula:

$$\delta^{*} = \frac{1}{1/\delta_h + 1/\delta_s - 1}\,,$$

where $\delta_h$, $\delta_s$ indicates emissivity of the radiating surfaces of the films. In addition, it has been taken into consideration that water is practically non-transparent for heat rays [*Mikheyev and Mikheyeva, 1977*].

To ensure the possibility of modeling on standard personal computers, we used a somewhat simplified mathematical description of heat exchange. It is based on consideration of air insulator as a thermal resistance $R$. A similar method is widely used in geocryological calculations to assess the insulating role of snow cover. In our case, the use of this method is even more justified, as the volumetric heat capacity of air is 3,500–4,000 times less than that of water. The amount of heat which raises the temperature of 5-cm thick air insulation by 1 °C may change the temperature of water in the sleeve (1 cm) by only 0.001 25–0.001 43 °C.

The value of $R$ was determined by the formulae known in thermal engineering [*Kutateladze, 1970; Mikheyev and Mikheyeva, 1977*] depending on the Prandtle (Pr) and Grashof criteria (Gr):

$$R = \frac{l}{0.18\lambda(\mathrm{PrGr})^{0.25}}\,, \tag{8}$$

where $\mathrm{Pr} = \nu/a$; $\mathrm{Gr} = g\beta\Delta T\frac{l^3}{\nu^2} = A\Delta T l^3$; $l$ is thickness of air insulation, m; $\lambda$ is thermal conductivity coefficient, W/(m·°C); $a$ is thermal diffusivity, $m^2/s$; $\nu$ is kinematic viscosity coefficient, $m^2/s$; $\beta$ is coefficient of volumetric air expansion; $g$ is gravity acceleration (9.81 $m/s^2$).

Formula (8) is correct if the condition PrGr >1000 is satisfied; at lower values of this product, the fraction's denominator in (8) is equal to $\lambda$. In the conditions considered, when the temperature difference is only 0.01 °C, the value of the PrGr complex > 3000.

Disregarding the temperature field of water and considering only the average water temperature, the calculation design will become much simpler. Such an approach fully suits the operation design of film heaters, in accordance with which all the water flows into a common tank and acquires common temperature.

It follows from the law of conservation of energy that at any moment of time the rate of total heat change $Q_w$ of the entire layer of water is equal to the difference between heat fluxes on its boundaries ($x = 0$ and $x = h + 0$). Hence, considering formulae (5), (6), we obtain

$$Q_w = \frac{T(s) - T(h)}{R} + \alpha_l(s-0)T(s) - \alpha_l(h)T(h) + Q_s(1-A). \qquad (9)$$

Thus, *each temperature of the water surface T(h) accords to a certain total heating rate*. This important conclusion allows a simplified algorithm of solving the problem to be applied without prejudice to precision. It consists in the following.

It is convenient to approximate the Stefan–Boltzmann thermal radiation law with the formula

$$I(T) = I_0 + \alpha_l(T)T,$$

where $I(T)$, $I_0$ is the density of thermal radiation of the surfaces having the temperature $T$ and 0 °C, respectively.

The temperature dependence of the approximation coefficients $\alpha_l$ of the resulting thermal radiation between the surfaces with their emissivity $\delta = 0.95$ is shown in Fig. 4. It is to be remembered that the emissivity of the outer surface ($x = s + 0$) of the air insulator has its own value $\delta$, whereas on the inner boundaries ($x = s - 0$ and $x = h$) reduced values $\delta^*$ are reached. Let on the outer surface of the film heater $\delta_s = 0.95$, then the value $I_0$ is 299.2 W/m$^2$. If the emissivity of a sleeve with water $\delta_h = 0.34$, then, in accordance with Christiansen's formula, the reduced emissivity $\delta^*(h) = \delta^*(s - 0) \approx 1/3$. In this case, $I_0(s + 0) = 157.48$ W/m$^2$ and $I_0(s - 0) = 104.99$ W/m$^2$.

With such an approach, the connection conditions are modified as follows. On the film surface ($x = s + 0$) separating the heater from atmosphere, a similar formula is set (7):

$$\frac{T(s) - T(h)}{R} + \alpha_l(s-0)T(s) - \alpha_l(h)T(h) = I_a - I_0(s+0) - \alpha_l(s+0)T(s) + \alpha_a\left[T_a - T(s)\right]. \qquad (10)$$

Hence we find the relation between $T(s)$ and $T(h)$:

$$T(s) = \frac{T(h)\left[R^{-1} + \alpha_l(h)\right] + I_a + \alpha_a T_a - I_0}{\alpha_l(s-0) + \alpha_l(s+0) + R^{-1} + \alpha_a}. \qquad (11)$$

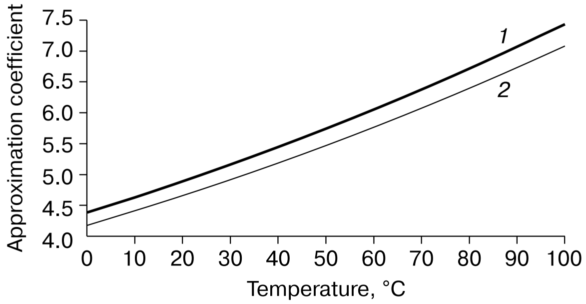


**Fig. 4. The approximation coefficient [W/(m$^2$·°C)] of the thermal radiation law for the outer (*1*) and inner (*2*) surfaces of an air insulator depending on temperature.**

## THE RESULTS OF CACULATIONS

When stationary water condition is reached, water heating stops. Subtracting (9) from (10) with $Q_w = 0$, we find the maximum values of $T(s)$ and $T(h)$:

$$T_{\max}(s) = \frac{Q_s(1-A) + I_a + \alpha_a T_a - I_0}{\alpha_a + \alpha_l(s-0)} = 36.3\,°\mathrm{C},$$

$$T_{\max}(h) = \frac{T_{\max}(s)\left[\alpha_l(s-0) + \alpha_l(s+0) + R^{-1} + \alpha_a\right] - I_a - \alpha_a T_a + I_0}{R^{-1} + \alpha_l(h)} = 78.1\,°\mathrm{C}.$$

The dependences shown allow us to determine the dynamics of heating, starting from any set temperature. For this purpose, a series of the temperature values $T(h)$ from the initial value to the maximum value is to be set. Then, using the formulae (8), (9) and (11) and iterations, we obtain the respective values of $Q_w$. After that, a supporting table of dependence $Q_w$ on $T(h)$ is composed. Identifying "quasi-linear areas" between two water temperature values $T(h)_n$ and $T(h)_{n+1}$, we find the mean value of heating intensity ($Q_m$):

$$Q_m = \frac{Q_w\left(T(h)_n\right) + Q_w\left(T(h)_{n+1}\right)}{2}.$$

Time $\Delta t_n$ of the increase in the average water temperature from $T(h)_n$ to $T(h)_{n+1}$ is calculated using the evident formula

$$\Delta t_n = c_w \rho_w h \frac{T(h)_{n+1} - T(h)_n}{Q_m},$$

where $c_w$, $\rho_w$, $h$ are specific heat, density, and thickness of the water layer, correspondingly.

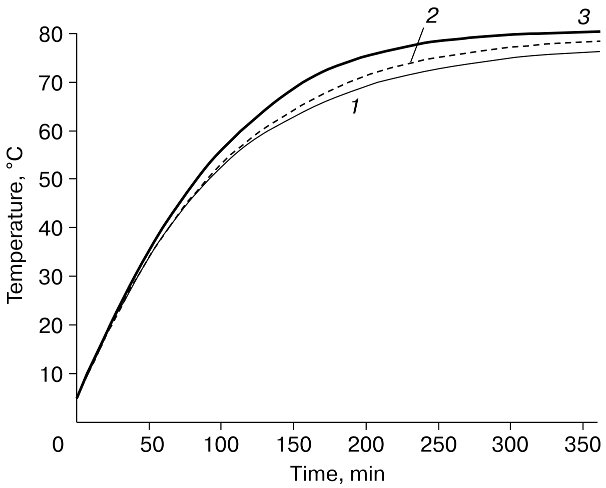


**Fig. 5. Dynamics of the averaged water temperature with different thickness values of air insulation (emissivity of the film surfaces 0.95).**

*1:* 2.5 cm; *2:* 7.5 cm; *3:* 15.0 cm.

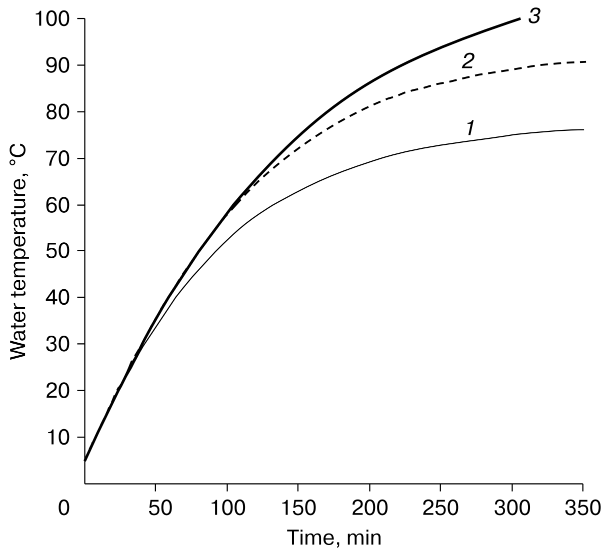


**Fig. 6. Water temperature in the heaters with different emissivity of the films ($l$ = 1 cm):**

*1:* 0.95; *2:* 0.75; *3:* 0.50.

The relations obtained allow the dependence of the dynamics of water heating on the primary parameters of the installation; thickness of the air insulation and the emissivity of the film, to be analized.

Shown on Fig. 5 is the dynamics of water heating, given air insulation with thickness $l$ varying from 2.5 to 15 cm (emissivity of the film surfaces is 0.95). With $l$ = 2.5 cm, the water temperature reaches 76.1 °C during six day hours, and with $l$ = 15 cm, it reaches 80.3 °C. Hence, within the indicated limits, the effect reached by increasing the amount of insulation proves to be relatively small.

A decrease in the emissivity, which may be achieved by processing the water sleeve surface with coating compositions, exerts higher influence on the water heating rate. In accordance with the reference data, for example, [*Mikheyev and Mikheyeva, 1977*], own values δ of inexpensive aluminum colorants are within the range of 0.3–0.6. It is possible to evaluate the reduced emissivity $\delta^*$ for a system consisting of two films: the upper nearly transparent film (δ = 0.95) and the lower film the value of δ of which is 0.5 due to coloring. Calculation by formula (6) yields the value of $\delta^* \approx 0.5$.

Fig. 6 clearly demonstrates the role of film emissivity in forming the mean water temperature. If $\delta^*$ = 0.905, then during 5 hours the water temperature rises to 75.5 °C. With $\delta^*$ = 0.75, water gets heated to 90 °C. And finally, in the system with reduced emissivity 0.5, water reaches the boiling point approximately within 5 hours.

In practice, water is often used, which is much colder than air. With the set parameters of "outer" heat exchange, the water temperature in the film water heaters considered will rise from 5 to 20 °C only within 23–24 minutes.

## CONCLUSION

In general, the results of the analysis made fully confirm the preliminary conclusion regarding the broad perspectives of applying simple film heaters for the purpose of hot water supply in the territory of the permafrost zone. The estimates made demonstrate that using environmentally clean film water heaters ensures an economic effect of about 3,000 rubles per one person per year. The application area for such heaters covers the entire country, except for large cities.

## References


Budyko, M.I., 1956. The Thermal Balance of the Earth's Surface. Gidrometeoizdat, Leningrad, 254 pp. (in Russian)

Butuzov, V.A., 2000. Solar heat supply: the status quo and the development prospects. Energosberezheniye, No. 4, 28–30.

Goldtman, V.G., Znamensky, V.V., Chistopolsky, S.D., 1970. Hydraulic thawing of permafrost rocks. Tr. VNII-1, Magadan, XXX, pp. 172–175. (in Russian)

Kurtener, D.A., Chudnovsky, A.F., 1969. Calculation and Control of Heat Transfer on Open and in Protected Ground. Gidrometeoizdat, Leningrad, 229 pp. (in Russian)

Kutateladze, S.S., 1970. The Foundations of the Heat Exchange Theory. Nauka, Novosibirsk, 660 pp. (in Russian)

Mikheyev, M.A., Mikheyeva, I.M., 1977. The Foundations of Heat Transfer. Energiya, Moscow, 341 pp. (in Russian)

Olovin, B.A., 1971. Heat exchange of thawing permafrost rocks and atmosphere in strip development of ground. Kolyma, No. 4, 10–12.

Pavlov, A.V., 1979. The Thermal Physics of Terrains. Nauka, Novosibirsk, 285 pp. (in Russian)

Pavlov, A.V., Perlshtein, G.Z., Tipenko, G.S., 2010. The topical aspects of modeling and forecastting the thermal condition of the permafrost zone under conditions of a changing climate. Kriosfera Zemli XIV (1), 3–12.

Perlshtein, G.Z., 2002. Heat exchange between active soil and atmosphere: theoretical and applied aspects. Kriosfera Zemli VI (1), 25–29.

Perlshtein, G.Z., Pavlov, A.V., Levashov, A.V., Sergeyev, D.O., 2005. Non-temperature factors of heat exchange between active soil and atmosphere. Materialy 3 Vserossiyskoy konf. po geokriologii. Moscow University Press, Moscow, Vol. 1, pp. 86–91.

*Received November 6, 2014*