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TEMPERATURE DISTRIBUTION IN SOIL COOLED BY AN HET THERMOSYPHON SYSTEM: PREDICTION BY STOCHASTIC ANALYSIS

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The operation of the system 'oil tank–HET thermosyphon–soil' is simulated for 576 random trajectories of air temperature, wind speed and snow depth changes over the past two years. The calculated probability distribution for the soil temperature pattern fits the normal law.

Monte Carlo simulation, thermosyphon, temperature field, temperature, stochastic analysis

INTRODUCTION

The performance of refrigeration systems based on natural convection (two-phase thermosyphons), with horizontal evaporator tubes (HET) [*Anikin et al., 2011*] can be evaluated by probabilistic prediction using stochastic analysis as suggested by *Spasennikova* [*2015*]. The procedure consists of several steps: (1) generating different scenarios of changes in air temperature, wind speed, and snow depth by Monte Carlo simulation; (2) calculating a 3D temperature field on a certain date for each scenario; (3) estimating the probability of finding unfrozen soil in presumably frozen areas. The number of condenser units in an HET cooling system is considered to be optimal at $0.1\,\% \leqslant w \leqslant 5\,\%$, insufficient at $w > 5\,\%$, and excessive at $w < 0.1\,\%$. In the latter case, this number can be reduced thus optimizing the operation, which saves costs of the HET system for the client. Note that this way of evaluating the system performance was suggested by the authors but clients can use their own way at the stage of design. The probability for soil to be unfrozen was estimated previously [*Anikin et al., 2013; Melnikov et al., 2014; Dolgikh et al., 2015*] assuming normal temperature probability distribution, which is however not obvious. In this study, the distribution of probable ground temperatures is calculated explicitly by Monte Carlo simulation of 576 random trajectories divided into 12 subgroups.

PROBLEM FORMULATION

The temperature pattern was simulated for soils beneath an oil tank cooled by an HET thermosyphon system at the Vankor oil field. The tank stores oil at 40 °C; the cooling system consists of ten condenser units with a finning area of 100 m^2 each, placed at a height of 3.83 m above the evaporator tubes that cover a circular area, 46 m in diameter, and are spaced at 0.5 m; the tank, 40 m in diameter, stands on a 0.28 m thick water-proof layer lying on 0.12 m of sand and 0.45 m thick penoplex (extrusive expanded polystyrene) at the base (layout same as in [*Anikin and Spasennikova, 2012*]).

In the modeling domain (Fig. 1), the evaporator tubes are parallel to the axis z and perpendicular to the axis y; the axis x is directed downward from the soil surface.

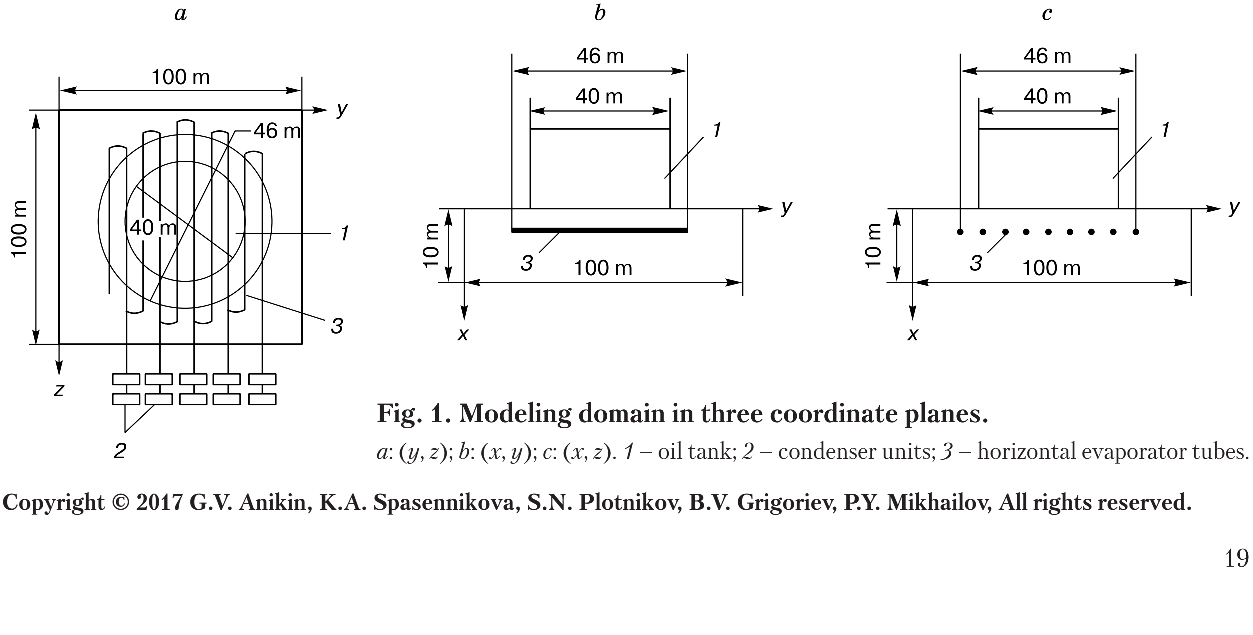

Fig. 1. Modeling domain in three coordinate planes.
a: (y, z); *b*: (x, y); *c*: (x, z). *1* – oil tank; *2* – condenser units; *3* – horizontal evaporator tubes.

Each random trajectory was simulated for a period of two years, from earliest September to latest August. Stochastic analysis was performed using the *Stohastic-3D* software. Its efficiency was confirmed by comparing predicted temperatures and field monitoring data at different objects [*Dolgikh et al., 2014; Spasennikova, 2015*]. The random trajectories were generated by a *Mathcad-14* random number generator. The modeling was performed on an *NCS-30T* supercomputer at the Siberian Supercomputing Center, in 12 runs each yielding simultaneously 48 3D temperature fields.

The probability distribution for the ground temperature is calculated, using 13 boundaries for 12 channels of the histogram, as

$$-17 + 1.5i,\ 0 \leqslant i \leqslant 12,$$

where i is the boundary number. The temperatures corresponding to the channel centers are given by

$$t_i = -16.25 + 1.5i,\ 0 \leqslant i \leqslant 11.$$

The temperature distribution is studied in detail at two points: points 1 and 2, at $x = 4.2$ m, $y = 50$ m, $z = 50$ m and $x = 0.7$ m, $y = 50$ m, $z = 50$ m, respectively, both on the central tank axis. The temperature histograms at these points for each m-th run and the sum of the histograms are presented in Tables 1 and 2, respectively.

The probability of finding a temperature lower than $t_i + 0.75$ (i-th upper channel boundary) for points 1 and 2 is

$$w_{1,i} = \frac{1}{576}\left(\sum_{k=0}^{i} n_{1,i}\right),\quad w_{2,i} = \frac{1}{576}\left(\sum_{k=0}^{i} n_{2,i}\right); \qquad (1)$$

$$\sum_{k=0}^{11} n_{1,i} = \sum_{k=0}^{11} n_{2,i} = 576 \rightarrow \sum_{k=0}^{11} w_{1,i} = \sum_{k=0}^{11} w_{2,i} = 1. \qquad (2)$$

The values $n_{1,i}$ and $n_{2,i}$ are given in the lower lines of Tables 1 and 2, respectively, while the values $w_{1,i}$ and $w_{2,i}$ calculated by (1) are given in Table 3, along with the temperature t_i.

Table 1. **Histograms for runs m and total distribution over all runs (last line) for point 1**

m \ i	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	2	14	10	16	5	1	0	0	0	0
1	0	1	2	5	16	15	8	1	0	0	0	0
2	0	0	4	5	16	14	9	0	0	0	0	0
3	0	1	2	7	9	16	9	4	0	0	0	0
4	0	1	3	13	11	10	6	4	0	0	0	0
5	1	0	6	7	12	15	7	0	0	0	0	0
6	0	1	4	5	16	16	4	2	0	0	0	0
7	0	0	5	7	11	20	4	1	0	0	0	0
8	0	1	5	4	15	15	7	1	0	0	0	0
9	1	0	7	10	10	14	5	1	0	0	0	0
10	0	1	3	9	10	18	6	1	0	0	0	0
11	0	1	1	6	16	13	8	3	0	0	0	0
$n_{1,i}$	2	7	44	92	152	182	78	19	0	0	0	0

Table 2. **Histograms for runs m and total distribution over all runs (last line) for point 2**

m \ i	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	1	6	15	9	10	5	1	1	0
1	0	0	0	2	5	11	8	15	6	1	0	0
2	0	0	0	3	4	9	11	12	9	0	0	0
3	0	0	2	1	6	5	14	8	8	4	0	0
4	0	0	1	3	5	8	13	7	7	4	0	0
5	1	0	0	5	7	4	16	9	5	1	0	0
6	0	1	1	1	4	6	19	10	4	1	1	0
7	0	0	0	4	5	9	9	16	4	1	0	0
8	0	0	0	4	5	10	8	14	6	1	0	0
9	0	1	0	4	9	6	9	12	6	1	0	0
10	0	0	1	2	6	12	6	16	4	1	0	0
11	0	0	1	0	4	8	13	11	8	2	1	0
$n_{2,i}$	1	2	6	30	66	103	135	140	72	18	3	0

Table 3. **Values t_i, $w_{1,i}$, $w_{2,i}$**

i	t_i, °C	$w_{1,i}$	$w_{2,i}$
0	–16.25	0.003 47	0.001 74
1	–14.75	0.015 63	0.005 21
2	–13.25	0.092 01	0.015 63
3	–11.75	0.251 74	0.067 71
4	–10.25	0.515 63	0.182 29
5	–8.75	0.831 60	0.361 11
6	–7.25	0.967 01	0.595 49
7	–5.75	1	0.838 54
8	–4.25	1	0.963 54
9	–2.75	1	0.994 79
10	–1.25	1	1
11	0.25	1	1

Table 4. **Values t_i, $w^n_{1,i}$, $w^n_{2,i}$**

i	t_i, °C	$w^n_{1,i}$	$w^n_{2,i}$
0	–16.25	0.001 59	0.000 29
1	–14.75	0.014 72	0.002 45
2	–13.25	0.079 83	0.014 46
3	–11.75	0.262 78	0.059 89
4	–10.25	0.554 33	0.176 99
5	–8.75	0.818 07	0.383 83
6	–7.25	0.953 47	0.629 60
7	–5.75	0.992 87	0.831 38
8	–4.25	0.999 36	0.943 91
9	–2.75	0.999 97	0.986 70
10	–1.25	0.999 99	0.997 79
11	0.25	0.999 99	0.999 74

The mathematical expectation $\overline{t}_1$ and the rms error σ_{t1} for point 1 are

$$\overline{t}_1 = \sum_{i=0}^{11} w_{1,i} t_i = -9.7656\,°\text{C},$$

$$\sigma_{t1} = \sqrt{\sum_{i=1}^{11} w_{1,i}\left(t_i - \overline{t}_1\right)^2} = 1.9445\,°\text{C}.$$

The mathematical expectation $\overline{t}_2$ and the rms error σ_{t2} for point 2 are

$$\overline{t}_2 = \sum_{i=0}^{11} w_{2,i} t_i = -7.2891\,°\text{C},$$

$$\sigma_{t2} = \sqrt{\sum_{i=1}^{11} w_{2,i}\left(t_i - \overline{t}_2\right)^2} = 2.3853\,°\text{C}.$$

The probability of finding a temperature lower than t for the normal distribution with the mathematical expectation $\overline{t}$ and the rms error σ_t is

$$w\left(t,\overline{t},\sigma_t\right) = \int_{-\infty}^{t} \frac{\exp\left(-\left(\tau-\overline{t}\right)^2 \big/ \left(2\sigma_t^2\right)\right)}{\sqrt{2\pi}\,\sigma_t} d\tau. \qquad (3)$$

For the upper channel boundaries $t_i + 0.75$, $0 \leqslant i \leqslant 11$,

$$w^n_{1,i} = w\left(t_i + 0.75, \overline{t}_1, \sigma_{t1}\right),\ w^n_{2,i} = w\left(t_i + 0.75, \overline{t}_2, \sigma_{t2}\right). \quad (4)$$

Calculations by (3) and (4) lead to values listed in Table 4.

The empirical and normal distributions of the probabilities for points 1 and 2 match perfectly (Fig. 2, *a*, *b*).

The distribution of probability for finding a temperature higher than t at points 1 and 2 (Fig. 3, *a*, *b*), as well as that shown in Fig. 2, fit the normal law. The probability of finding a temperature above 0.25 °C at point 2 (Table 4) is $7.4 \cdot 10^{-4}$, i.e., the cooling system has too many condenser units and their number should be reduced. This result is identical to that of *Spasennikova* [*2015*].

In order to check whether this information can be obtained in a single run, the normal distributions of each run are compared with that of the whole statistical population (see compared runs 1 and 11 in Fig. 4).

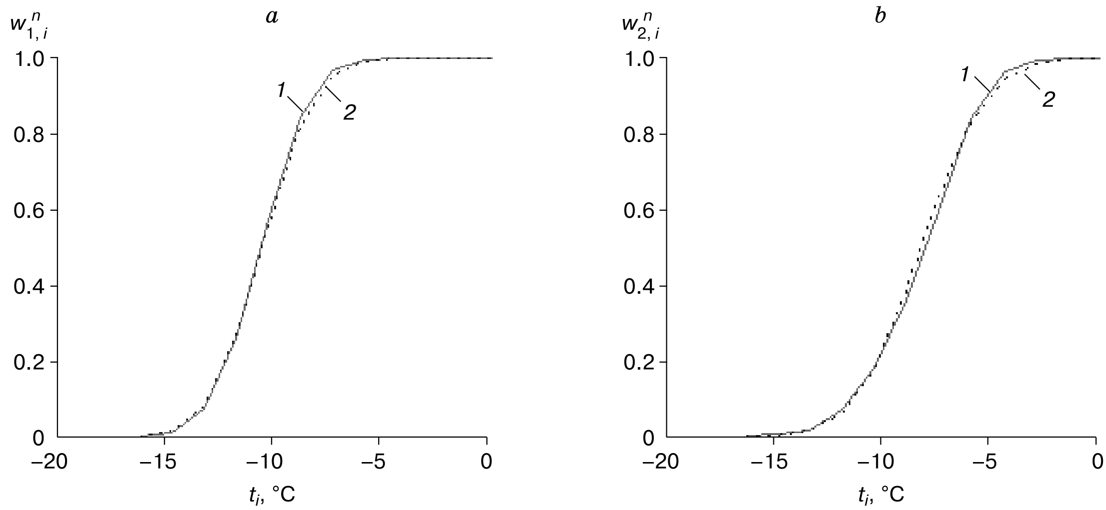

Fig. 2. Empirical (*1*) and normal (*2*) distributions of probability for finding a temperature lower than the specified value, compared (*t*), at points 1 (*a*) and 2 (*b*).

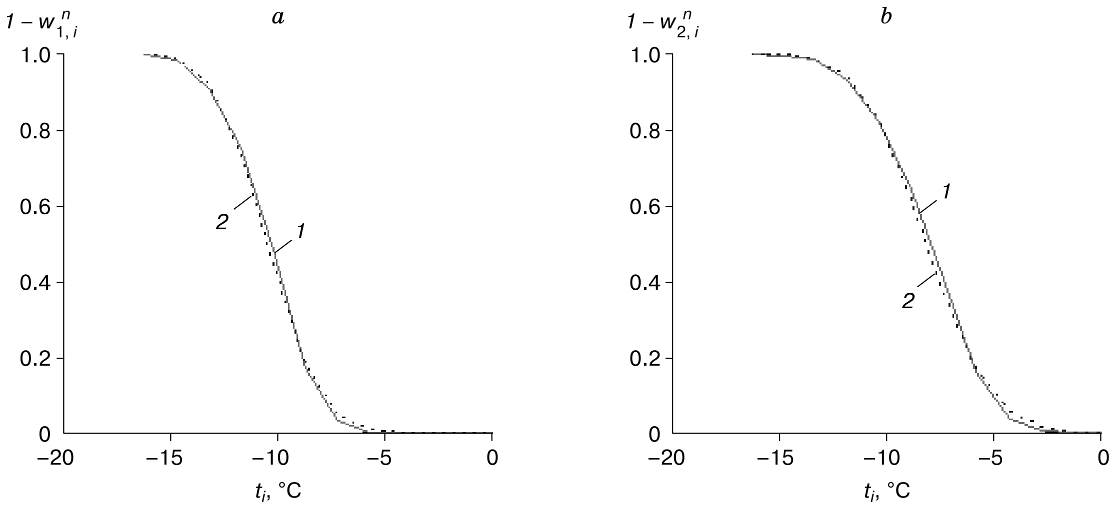

Fig. 3. Empirical (*1*) and normal (*2*) distributions of probability for finding a temperature higher than the specified value, compared (*t*), at points 1 (*a*) and 2 (*b*).

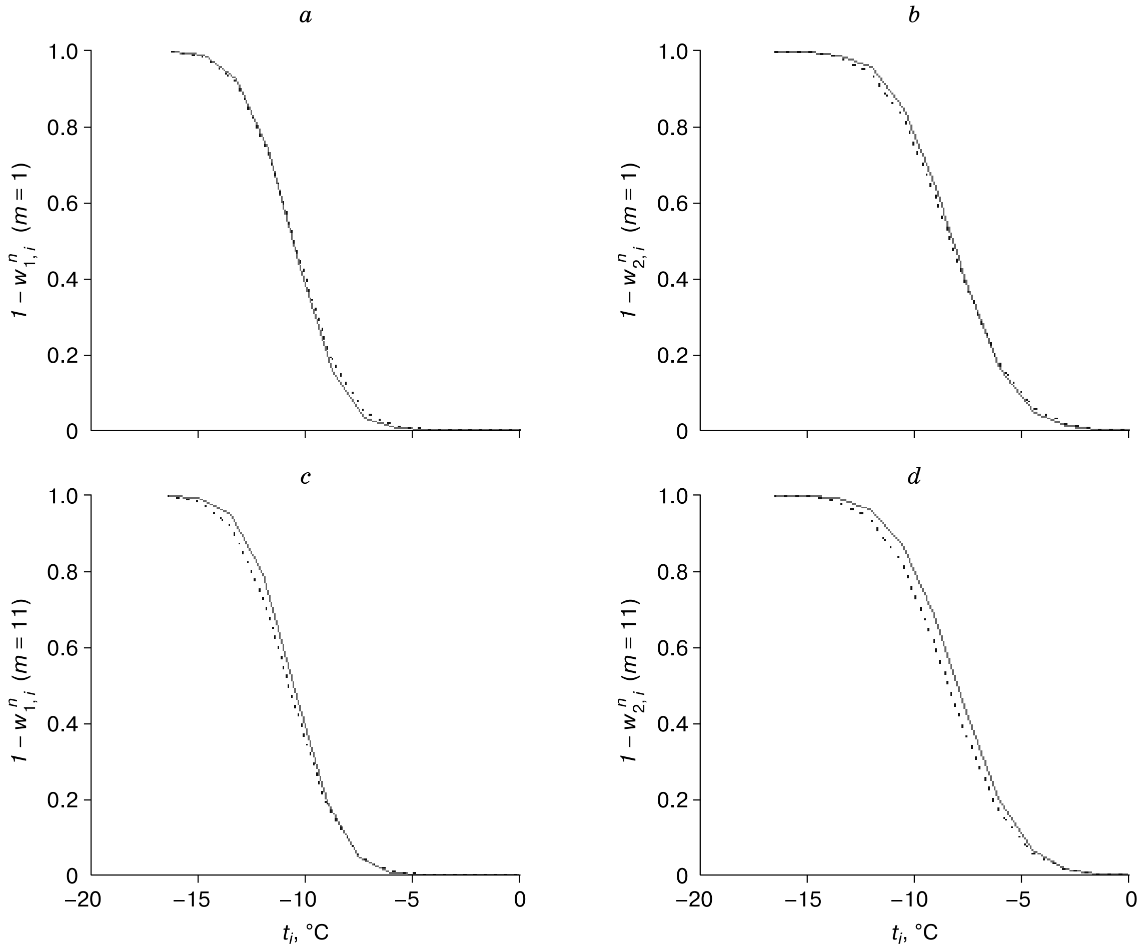

Fig. 4. Probability for finding a temperature higher than the specified value at points 1 (*a*, *c*) and 2 (*b*, *d*): normal distributions of run with *m* = 1 and 11 (solid lines) and the total of all runs (dash lines), compared.

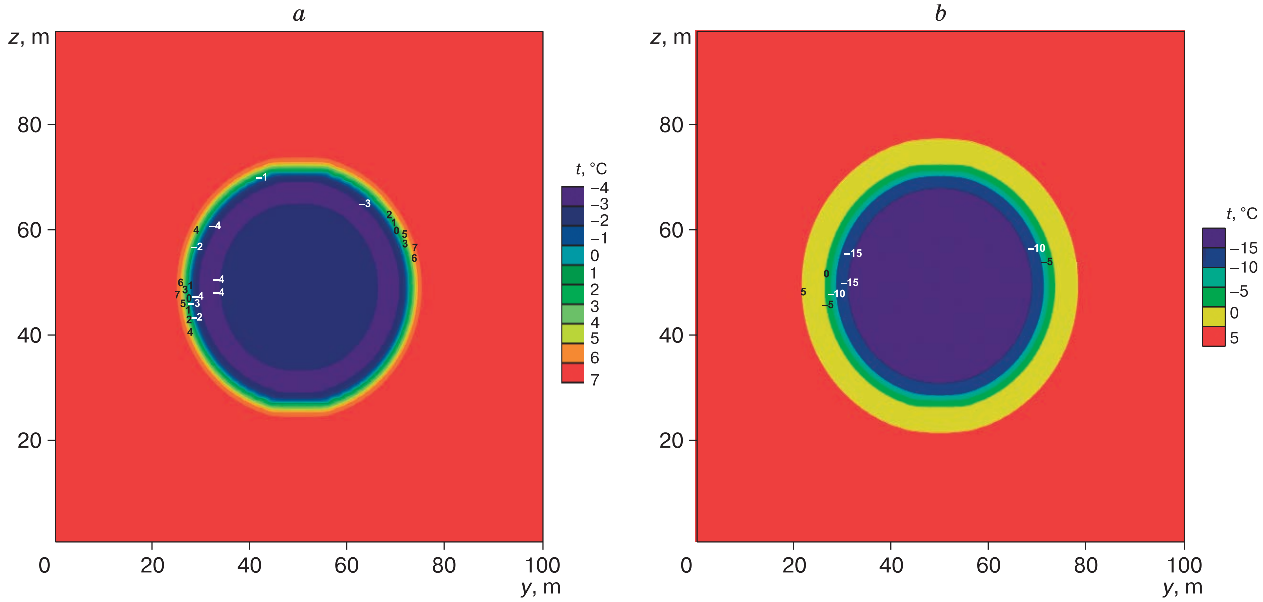

Fig. 5. Ground temperature field at the level of evaporator tubes (1.25 m below penoplex).

a: warmest scenario (n = 39), run m = 0; *b*: coldest scenario (n = 15), run m = 5.

The distribution of a single run is very similar to that for the total of all runs (Fig. 4), which allows estimating the performance of HET systems from information of one run (48 random trajectories).

As an illustration, Fig. 5 shows temperature fields in the end of August of the second modeling year for the warmest and coldest scenarios in the plane of the evaporator tubes.

CONCLUSIONS

The operation of HET thermosyphon has been predicted by stochastic analysis for 576 random trajectories of weather parameters. This number of random trajectories allowed calculating the probability of ground temperature patterns. The calculated probability distributions fit the normal law. The distribution for the total of 12 runs matches perfectly those for each single run consisting of 48 random trajectories. Therefore, information from one run (48 random trajectories generated by the Monte Carlo method) is sufficient to assess the performance of HET systems.

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